EXAM SIMULATION

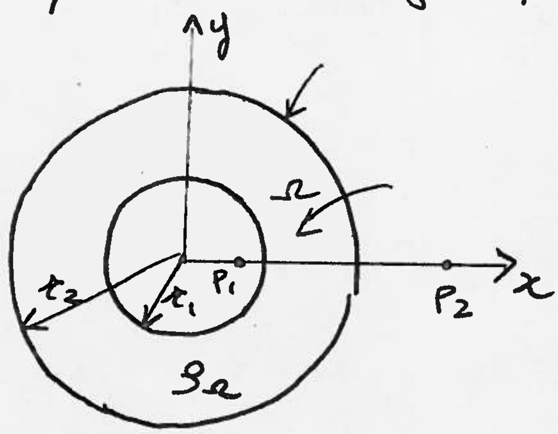
EXERCISE 1 (Electrostatics)

Let's consider the volume charge distribution shown in the picture, with $\rho_\Omega = 10^{-9}\,C/m^3$ in the region $r_1 \le r \le r_2$ ($r_1 = 10$ cm, $r_2 = 20$ cm)

Calculate the force experienced by a point charge $q = 2\,C$ in

a) P_1 (8 cm, 0)

b) P_2 (40 cm, 0)



The electric field generated by the charge distribution can be calculated by using the Gauss law

a) in P_1 (8 cm, 0) $\Rightarrow$ $r < r_1$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{S} = \int_\Omega \rho_\Omega \, d\Omega = \underset{\substack{\rho_\Omega = 0 \\ (r < r_1)}}{\cdots\cdots} = 0 \qquad E(r) = 0$$

$\Rightarrow$ in P_1 the force is zero

b) in P_2 (40 cm, 0) $\Rightarrow$ $r > r_2$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{S} = \varepsilon_0 E(r)\, 4\pi r^2 = \frac{4}{3}\pi \rho_\Omega (r_2^3 - r_1^3)$$

$$\vec{E}(r) = \frac{\rho_r}{3\varepsilon_0} \frac{r_2^3 - r_1^3}{r^2} \vec{a}_r$$

in $P_2 \Rightarrow \vec{E}(P_2) = \dfrac{10^{-9} C m^{-1} (0{,}2^3 - 0{,}1^3) m^3}{8{,}85 \cdot 10^{-12} F m^{-1}\ 3 \cdot 0{,}4^2\ m^2}$

$$= 1{,}65\ \vec{a}_x\ [V/m]$$

$$\vec{F} = \vec{E}(P_2) q = 3{,}3\ \vec{a}_x\ [N]$$

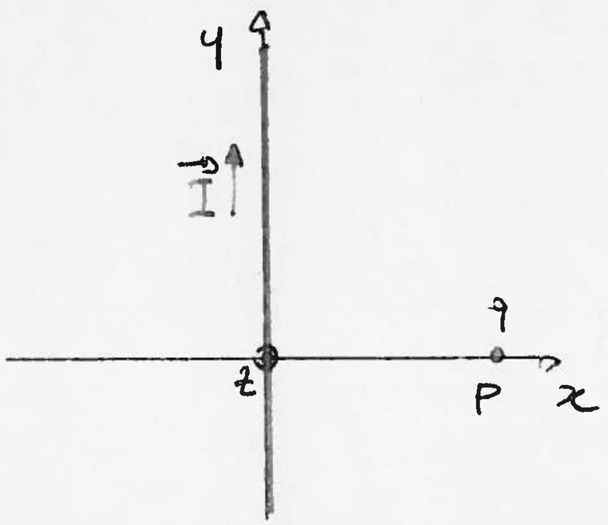
EXERCISE 2 (Magnetostatics)

A metallic wire, overlapped to the y axis, carries a current $\vec{I} = 3{,}14\ \vec{a}_y$ [A]. Calculate the force (vector) experienced by a point charge $q = 2C$ located in $P(1\,m, 0)$ when

a) q is at rest

b) q has a speed $v_y = 10$ Km/h

c) q " $v_z = 20$ Km/h



A charge moving in a magnetic field $\vec{B}$ experiences a force (Lorentz force) given by

$$\vec{F}_L = q\ \vec{v} \times \vec{B}$$

The magnetic field generated by the wire is

$$\vec{B} = \mu_0 \frac{I}{2\pi r} \vec{a}_\varphi$$ that in point P is

$$\vec{B}(P) = \mu_0 \frac{3{,}14\ A}{2\pi \cdot 1\,m} = 6{,}28 \cdot 10^{-7} (-\vec{a}_z)\ [T]$$

a) If q is at rest, $\vec{v} = 0 \implies \vec{F}_L = 0$

b) If q moves with $v_y = 10\ Km/h\ (= 2{,}78\ m/s)$

$$\vec{F}_L = q\, \vec{v}_y \times \vec{B} = 2C \cdot 2{,}78\ m s^{-1} \cdot 6{,}28 \cdot 10^{-7}\ T$$
$$= -3{,}5 \cdot 10^{-6} \vec{a}_x\ [N]$$

c) If q moves with $v_z = 20\ Km/h$

$$\vec{v}_z \parallel \vec{B} \implies \vec{F}_L = 0$$

EXERCISE 3 (Lossy transmission lines)

A coaxial cable having conductors with diameters $b = 10\,mm$ and $a = 2{,}5\,mm$ $(\sigma_c = 37{,}7 \cdot 10^6\ S/m)$ is filled with a dielectric with $\varepsilon_r = 4$

a) Calculate the attenuation [dB/Km] at a frequency $f = 100\ MHz$

b) What characteristics should the dielectric have in order to halve the loss without changing the other parameters of the line?
geometric or phisical

The attenuation is given by $\alpha = \dfrac{r}{2 Z_c}$ where $Z_c = \sqrt{\dfrac{\ell}{c}} = \dfrac{\sqrt{\varepsilon\mu}}{c}$

For a coaxial cable

$$c = \frac{2\pi\varepsilon}{\ln(b/a)} \Rightarrow Z_c = \frac{\sqrt{\varepsilon\mu}\,\ln(b/a)}{2\pi\varepsilon} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\ln(b/a)}{2\pi\sqrt{\varepsilon_r}}$$

$$= 41{,}6\ [\Omega]$$

The resistance per unit length

$$r = r_a + r_b = \frac{1}{\sigma\delta\pi}\left(\frac{1}{a} + \frac{1}{b}\right) = 0{,}515\,[\Omega/m]$$

where $\delta = \sqrt{\dfrac{2}{\omega\mu_0\sigma}} = 8{,}2\,\mu m$

$$\alpha = \frac{r}{2 Z_c} = 6{,}2 \cdot 10^{-3}\ Np/m = 53{,}8\ dB/Km$$

$\times 8686$

b)

$$\alpha = \frac{r}{2Z_c} = \frac{1}{\sigma\delta\pi}\left(\frac{1}{a}+\frac{1}{b}\right)\frac{2\pi\sqrt{\varepsilon_r'}}{2\sqrt{\frac{\varepsilon_0}{\mu_0}}\ln\left(\frac{b}{a}\right)}$$

$$= \frac{1}{\sigma\delta}\left(\frac{1}{a}+\frac{1}{b}\right)\frac{1}{\sqrt{\frac{\varepsilon_0}{\mu_0}}\ln\left(\frac{b}{a}\right)}\sqrt{\varepsilon_r'}$$

The attenuation constant scales with $\sqrt{\varepsilon_r'}$, so a material with $\varepsilon_r = 1$ (air) will have

$$\alpha' = \frac{\alpha}{\sqrt{\varepsilon_r'}} = \frac{\alpha}{2} = 26{,}9 \text{ dB/Km}$$

Note that the characteristic impedance will change as well

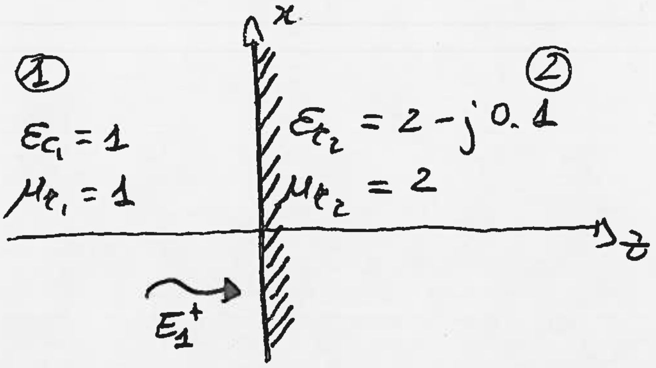
$$Z_c' = Z_c\sqrt{\varepsilon_r'} = 83.2\ \Omega$$

EXERCISE 4 (Plane waves)

A uniform plane wave propagating at a frequency $f = 300\,MHz$ along the direction $+z$ in air ($\varepsilon_{r_1} = 1$, $\mu_{r_1} = 1$) impinges normally onto a lossy dielectric ($\varepsilon_{r_2} = 2 - j0.1$, $\mu_{r_2} = 2$)

The forward electric field at the interface is $\vec{E}_i(0,0,0) = 1\,\vec{a}_x$ [V/m]

Calculate the magnetic field $\vec{H}_2$ in $z = \lambda_2$



For good dielectrics the intrinsic impedance of the medium is $\eta = \sqrt{\dfrac{\mu}{\varepsilon}}$

$$\Rightarrow \quad \eta_1 = 377\,\Omega$$

$$\eta_2 = \sqrt{\frac{\mu_{r_2}}{\varepsilon_{r_2}}}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{2}{2-j0.1}} \cdot 377\,\Omega \simeq 377\,\Omega$$

$$\hookrightarrow \operatorname{tg} \phi_2 = \frac{\varepsilon_{r_2}''}{\varepsilon_{r_2}'} = \frac{0.1}{2} \ll 1$$

This means that reflections at the interface is negligible

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \simeq 0 \quad \Rightarrow \begin{cases} E_r = 0 \\ E_{t_0} = E_i \end{cases}$$

The general expression of the electric field in medium 2 is

$$\vec{E}_2(z) = E_{t_0}\, e^{-\gamma_2 z} = E_i\, e^{-\alpha_2 z} e^{-j\beta_2 z}$$

At a distance $z = \lambda_2$ the phase shift of the wave

$$\beta_2 z \Big|_{z=\lambda_2} = \beta_2 \lambda_2 = \frac{2\pi}{\lambda_2}\lambda_2 = 2\pi \quad \Rightarrow e^{-j\beta_2 \lambda_2} = 1$$

Attenuation α_2 is

$$\alpha_2 = Re\left[\sqrt{j\omega\mu_2 j\omega\,\varepsilon_2}\right] = Re\left[j\omega\sqrt{\mu_2\varepsilon_2}\right] = 0.314 \quad [Np/m]$$

From the expression of $\beta_2 = \frac{2\pi}{\lambda_2}$, we derive the wavelength in medium 2

$$\lambda_2 = \frac{2\pi}{\beta_2} = \frac{2\pi}{Im\left[j\omega\sqrt{\mu_2\varepsilon_2}\right]} = 0.5\,m$$

At a distance $z = \lambda_2$ the transmitted electric field is

$$\vec{E}_2(\lambda_2) = 1\,e^{-\alpha_2\lambda_2}\,\vec{a}_x = 0{,}855\,\vec{a}_x$$

The magnetic field is

$$\vec{H}_2(\lambda_2) = \frac{|\vec{E}_2(\lambda_2)|}{\eta_2}\,\vec{a}_y = 0{,}0023\,\vec{a}_y$$

directed in the positive y direction

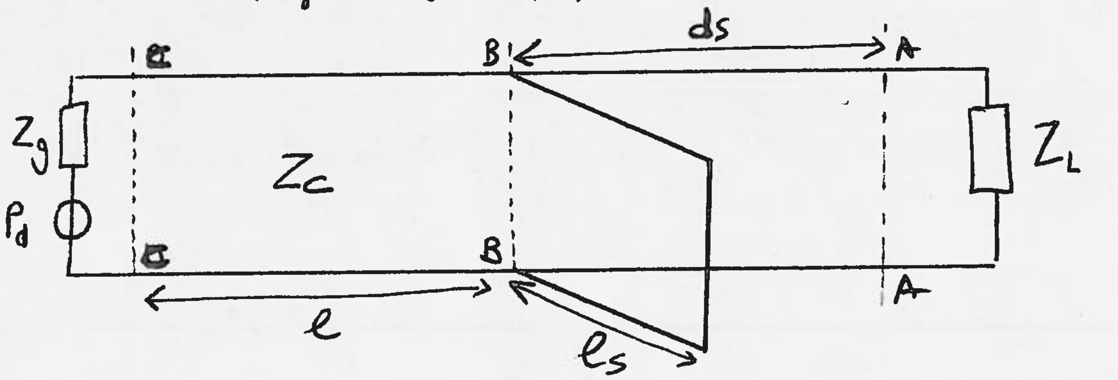
EXERCISE (Impedance matching and power dissipation)

A generator with frequency $f = 300$ MHz, internal impedance $Z_g = 80\,\Omega$ and voltage $V_g = 50$ V is connected with a load $Z_L = 50 + j\,10\,\Omega$ through a lossless ($\varepsilon_r = 1$) transmission line with characteristic impedance $Z_c = 80\,\Omega$ and length ℓ (see picture). Calculate

1) power dissipated by the load <u>without</u> the matching network for $\ell = 0.5$ m and $\ell = 0{,}25$ m

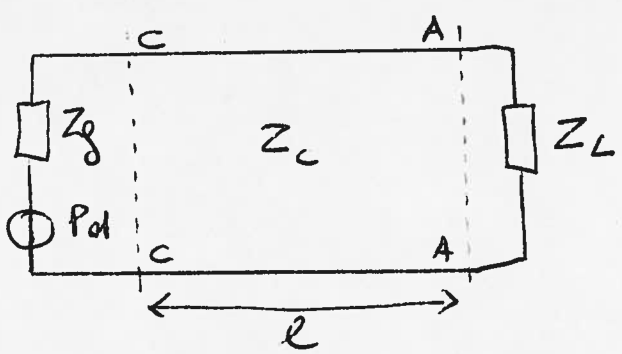
2) design the parallel stub network (short circuit) in order to match the load to the line

3) power dissipated by the load <u>with</u> the macthing network for $\ell = 1$ m



$f = 300$ MHz
$V_g = 50$ V
$\varepsilon_r = 1$

1) without the matching network



$Z_g = 80\,\Omega$
$Z_c = 80\,\Omega$
$Z_L = 50 + j\,10\,\Omega$

$Z_g = Z_c \Rightarrow \Gamma_g = 0$

$$\Gamma_{AA} = \frac{Z_L - Z_c}{Z_L + Z_c} = -0{,}224 + j\,0.094 = \Gamma_L$$

$$P_L = P_{AA}^+ (1 - |\Gamma_L|^2) = \dots \quad \text{no loss}$$

$$= P_{BC}^+ (1 - |\Gamma_L|^2) = \dots\dots \quad \text{line matched to the generator}$$

$$= P_d (1 - |\Gamma_L|^2) = 3,9 \text{ W} (1 - |0,2425|^2) = 3,68 \text{ W}$$

$$P_d = \frac{1}{8} \frac{V_g^2}{Z_g} = 3,9 \text{ W}$$

$\Rightarrow$ P_L does <u>not</u> depend on ℓ

2) For the matching network, let's use transmission line sections with $Z_c = 80\,\Omega$

Since the circuit is based on a <u>parallel</u> stub it is more convenient to work with admittance

$$\bar{y}_L = \frac{Y_L}{Y_c} = \frac{Z_c}{Z_L} = 1,538 - j\,0,308$$

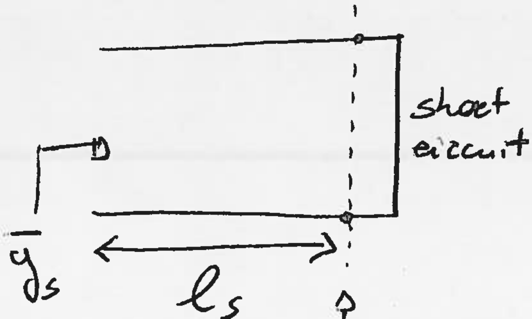
The length of the line section d_s bringing the admittance to the point

$\bar{y}_s' = 1 - j0.5$ is

$$\frac{d_s}{\lambda} = 0,355 - \cancel{0,0??}\,0,281 = 0,074 \qquad d_s = 0,074\,\lambda$$

$$\lambda = \frac{v}{f} = \frac{3 \cdot 10^8}{3 \cdot 10^8} = 1 \text{ m} \qquad d_s = 7,4 \text{ cm}$$

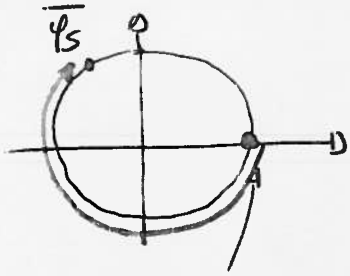
Design of the stub line



at this section $\bar{y} = \infty$

Condition $\bar{y}' + \bar{y}_s = 1$

$\Rightarrow \bar{y}_s = j0.5$



short circuit in the Smith Chart of <u>admittance</u>

$$\frac{l_s}{\lambda} = 0,25 + 0,074 = 0,324$$

$$l_s = 0,324\,\lambda = 32,4 \text{ cm}$$

3) With the matching circuit the full power available at the generator is dissipated by the load

$$P_L = P_d = 3,9 \text{ W}$$

Smith Chart

(of admittance