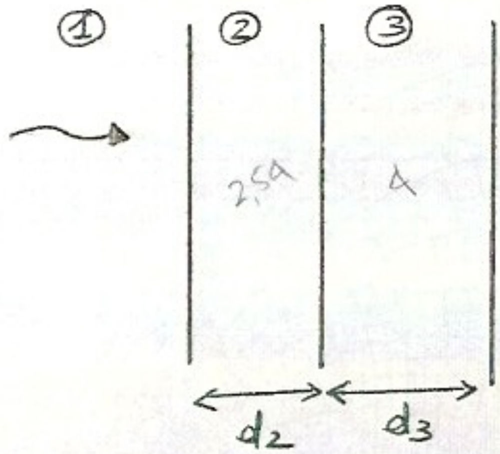
EXERCISE (Reflection on a multilayered dielectric)

Calculate the power reflection and transmission for the structure shown in the picture



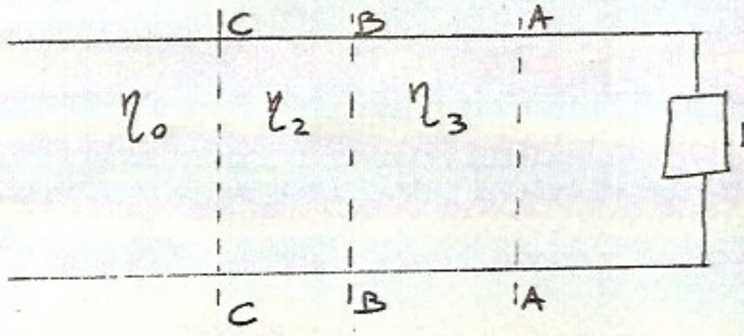
$\varepsilon_{r_1} = 1$ $\quad \varepsilon_{r_2} = 2.54$ $\quad \varepsilon_{r_3} = 4$

$d_2 = 2mm$ $\quad d_3 = 3mm$

$f = 10\ GHz$ $(\lambda_0 = 3\ cm)$

$\lambda = \frac{c}{f} = 3\ cm$

Let's consider the equivalent transmission line



$\eta_0 = 377\ \Omega$

$\eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = 236,5\ \Omega$

$\eta_3 = \frac{\eta_0}{\sqrt{\varepsilon_{r_3}}} = 188,5\ \Omega$

To calculate the reflection coefficient $\Gamma_{CC'}$ we start from the load section A and move back toward the "generator"

Each section of the line has the following "electrical" length (geometric length normalized to λ)

$$\frac{d_2}{\lambda_2} = \frac{2 \cdot 10^{-3} \sqrt{2,54}}{3 \cdot 10^{-2}} = 0,106$$

$$\frac{d_3}{\lambda_3} = \frac{3 \cdot 10^{-3} \sqrt{4}}{3 \cdot 10^{-2}} = 0,2$$

The normalized impedance at section A is

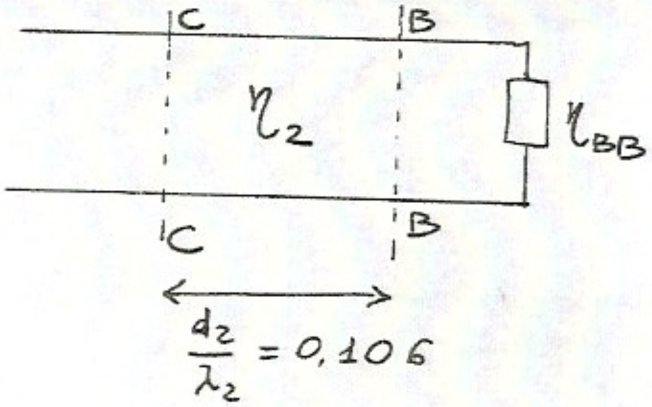
$$\bar{\eta}_{AA} = \frac{\eta_A}{\eta_3} = 2$$

The normalized impedance at section B is obtained by rotating point $\bar{\eta}_{AA}$ around the origin of the Smith Chart for a length d_3/λ_3

$\bar{\eta}^{(3)}_{BB} = 0,53 - j\,0,24$ (normalized to η_3)

$$\eta_{BB} = \bar{\eta}_{BB}\,\eta_3 = 100 - j\,45\ \Omega$$

The circuit is now equivalent to the one shown here below



Let's normalize η_{BB} to η_2 $= 236,5\Omega$

$\bar{\eta}^{(2)}_{BB} = \dfrac{\eta_{BB}}{\eta_2} = 0,42 - j\,0,19$ (normalized to η_2)

To move from section B to section C, rotate the point $\bar{\eta}_{BB}$ around the origin of the Smith Chart for a length $d_2/\lambda_2 = 0,106$

$\bar{\eta}_{cc} = 0,485 + j0,38$ (normalized to η_2)

$\eta_{cc}^{(2)} = \bar{\eta}_{cc}\,\eta_2 = 114.7 + j90 \;\Omega$

The reflection coefficient at section C is simply given by

$$\Gamma_c = \frac{\eta_{cc} - \eta_0}{\eta_{cc} + \eta_0} = -0,484 + j0,27 = 0,555\, e^{j2,63}$$

$$\measuredangle \Gamma_c = 150,7^\circ$$

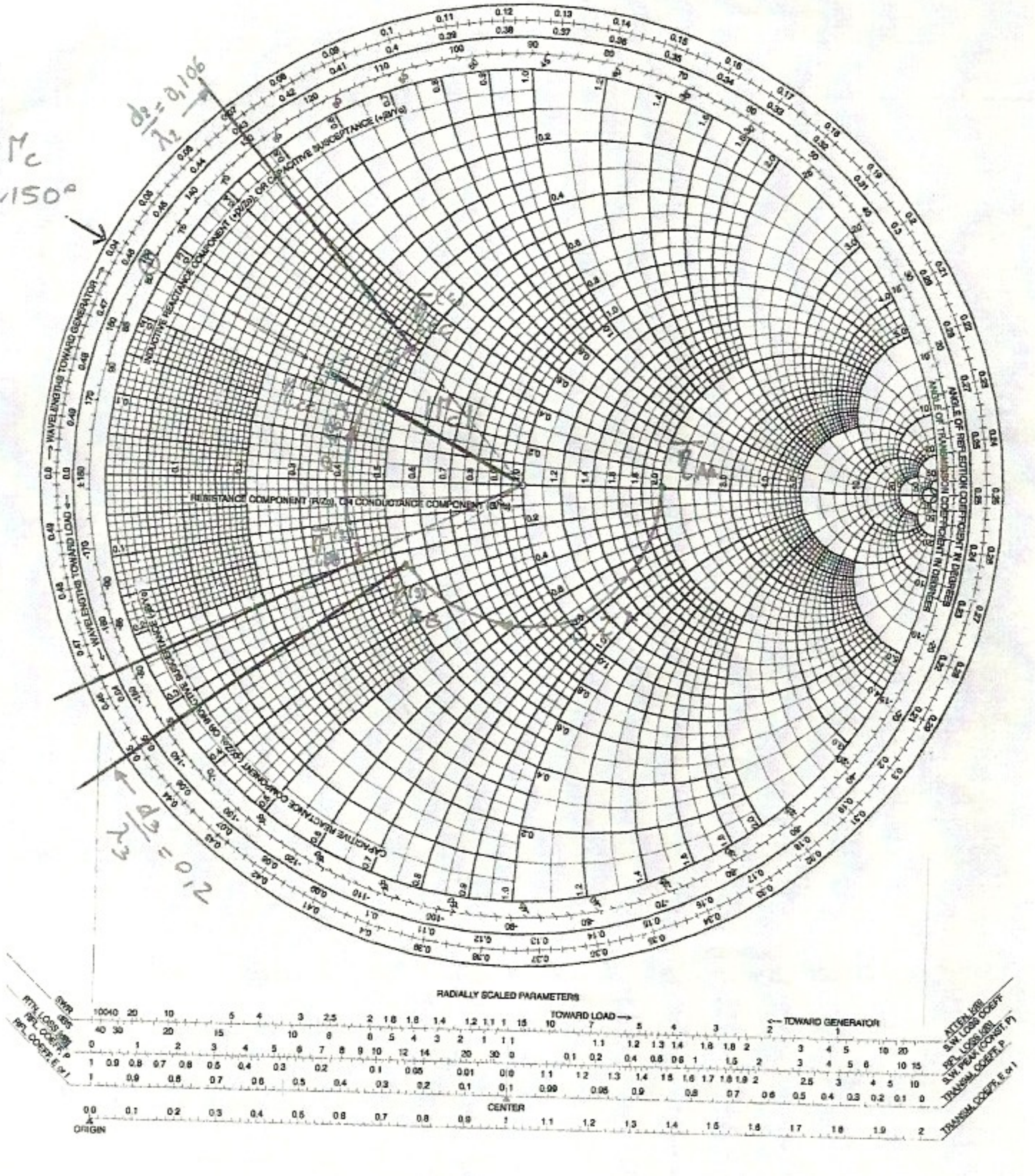
<u>Note</u> Γ_c can be also evaluate by normalizing η_{cc} to η_0 (see Smith Chart) and measuring $|\Gamma_c|$ and $\measuredangle \Gamma_c$ directly onto the chart $\eta_{cc}^{(0)} = 0,3 + j0,24$

The reflected power $\frac{P_R}{P_{in}} = |\Gamma|^2 = 0,305$ (30.5%)

The transmitted power (lossless medium) is

$$\frac{P_T}{P_{in}} = 1 - |\Gamma|^2 = 0,695 \quad (69.5\%)$$

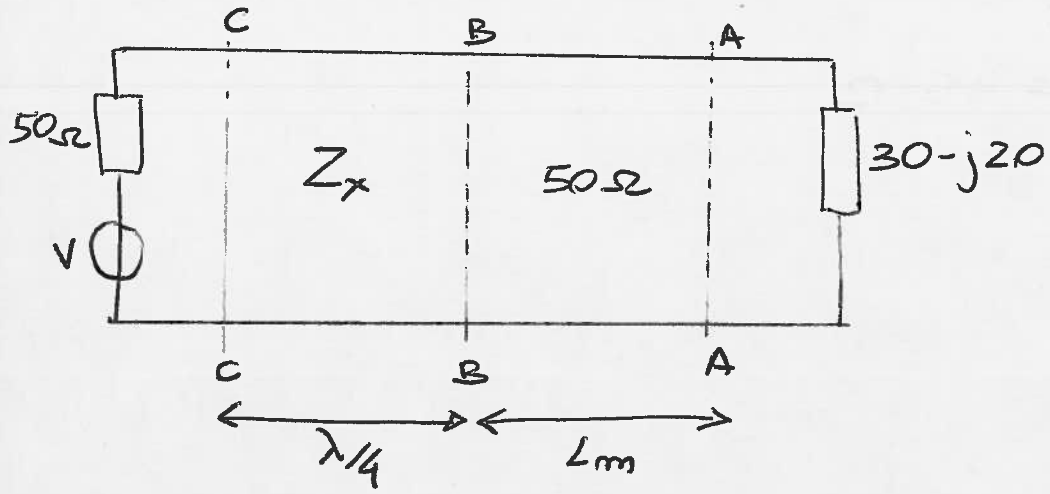
Smith Chart



RADIALLY SCALED PARAMETERS
TOWARD LOAD ⟶
⟵ TOWARD GENERATOR
CENTER
ORIGIN
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
$\measuredangle\Gamma_c$ ~150°
$\frac{d_2}{\lambda_2}$ = 0,106
$\frac{d_3}{\lambda_3}$ = 0,2

EXERCISE

For the transmission line shown in the picture, find the values of Z_x and L_m providing impedance matching at the generator section ($f = 1\,GHz$, $\lambda = 30\,cm$)



The transmission line section Z_x can provide impedance matching only if the impedance of the line at section B is real.

The normalized impedance at section A is

$$\bar{z}_{AA} = \frac{Z_{AA}}{50\Omega} = 0,6 - j0.4$$

In order to have a real impedance at section B the length L_m must be responsible for a rotation around the origin of the Smith Chart up to a crossing with the real axis.

A rotation to the point $\bar{z}_{BB} = 0.49$ is obtained when $L_m = 0.082\,\lambda = 2,46\,cm$

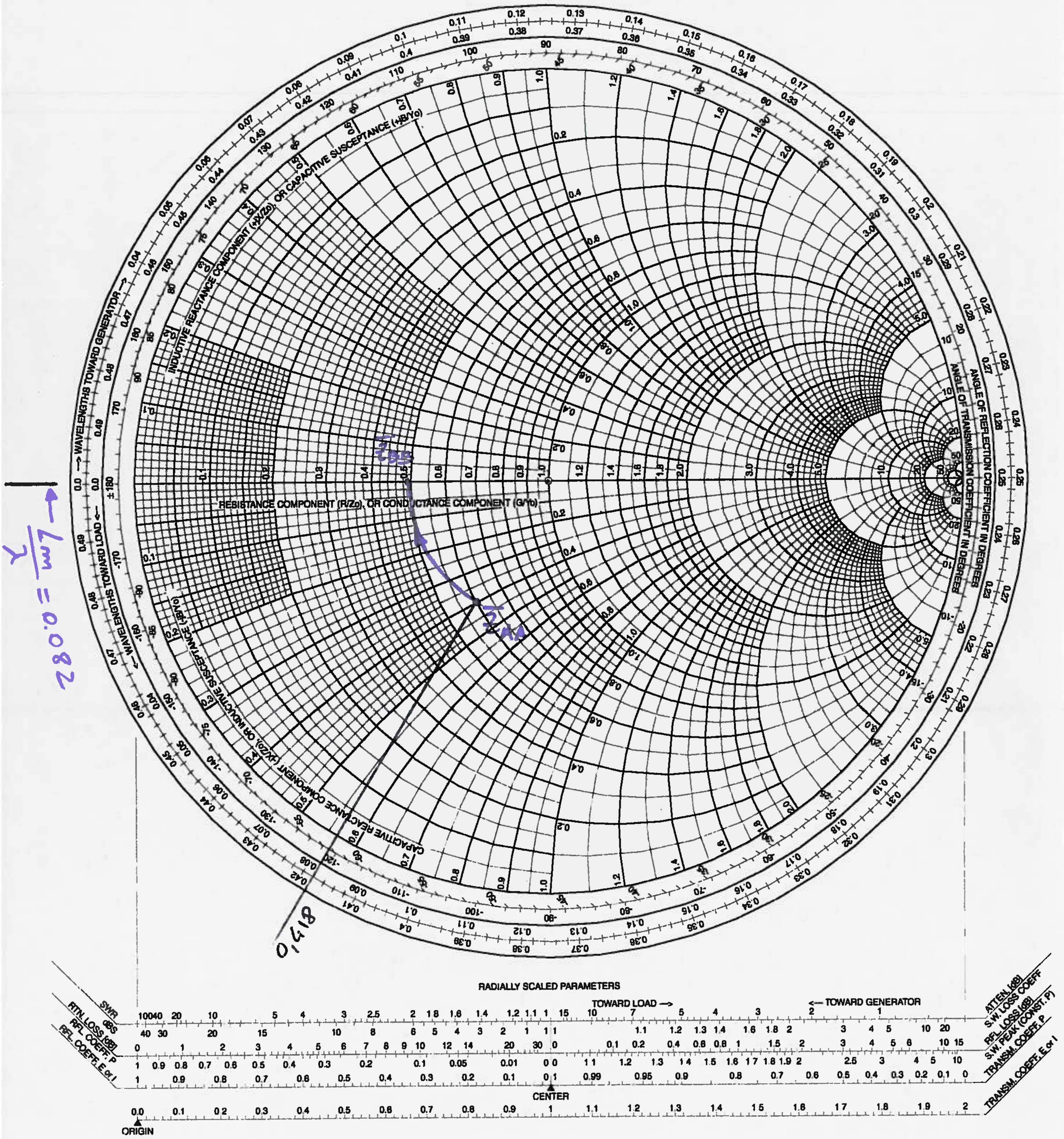
The impedance Z_{BB} is equal to

$$Z_{BB} = \bar{z}_{BB} \cdot 50\,\Omega = 24.5\,\Omega$$

The required characteristic impedance Z_x for the $\lambda/4$ matching section is

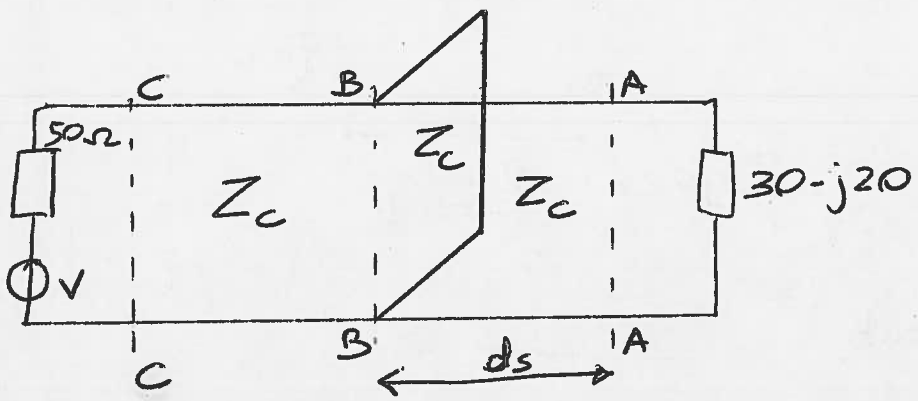
$$Z_x = \sqrt{50 \cdot Z_{BB}} = 35\,\Omega$$

Smith Chart


WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
0,418
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
SWR
dBS
RTN. LOSS (dB)
RFL. COEFF. P
RFL. COEFF. E or I
ATTEN. (dB)
S.W. LOSS COEFF
RFL. LOSS (dB)
S.W. PEAK (CONST. P)
TRANSM. COEFF. P
TRANSM. COEFF. E or I
CENTER
ORIGIN

EXERCISE (Impedance matching with stub line)

Calculate the length of the lines d_s and ℓ_s in order to have perfect impedance matching at the generator section for the line shown in the picture.



$Z_c = 50\,\Omega$

$f = 1\,\text{GHz}$

$\lambda = 30\,\text{cm}$

To design an impedance matching stub line in parallel to the main line, it is convenient to work with the Smith chart for admittance

The normalized admittance at section A is

$$\bar{y}_{AA} = \frac{Y_{AA}}{Y_C} = \frac{Z_C}{Z_{AA}} = \frac{50\,\Omega}{30 - j20\,\Omega} = 1{,}15 + j\,0.77$$

The line section d_s must move the point $\bar{y}_{AA}$ to a point with $Re\{\bar{y}_{BB}\} = 1$

Rotating $\bar{y}_{AA}$ around the center of the Smith chart, the first solution we find is

$$\bar{y}_{BB} = 1 - j\,0{,}73$$

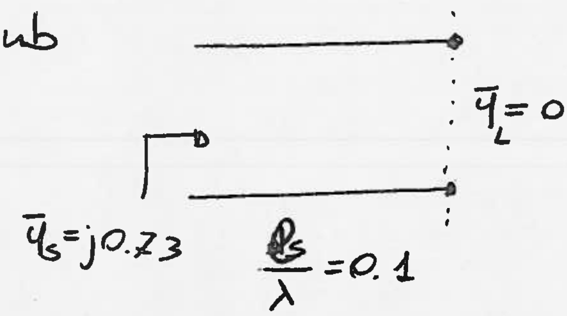
The length d_s is given by $\frac{d_s}{\lambda} = 0.348 - 0{,}168 = 0{,}18\,\lambda$

$d_s = 0{,}18\,\lambda = 5{,}4$ cm

The stub line must add an admittance $\bar{y}_s = +j0.73$ in order to cancel out the imaginary part of $\bar{y}_{BB}$

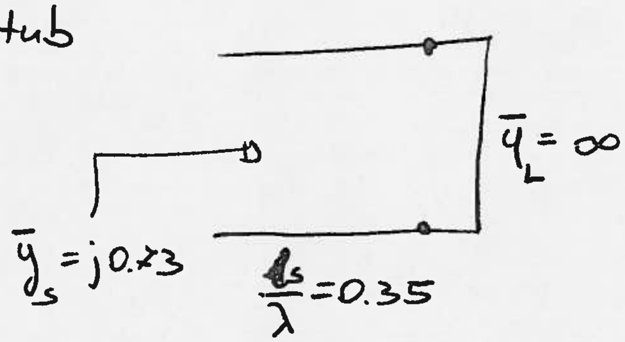
By using an open circuit stub

$d_s = 0.1\,\lambda = 3$ cm



By using a short circuit stub

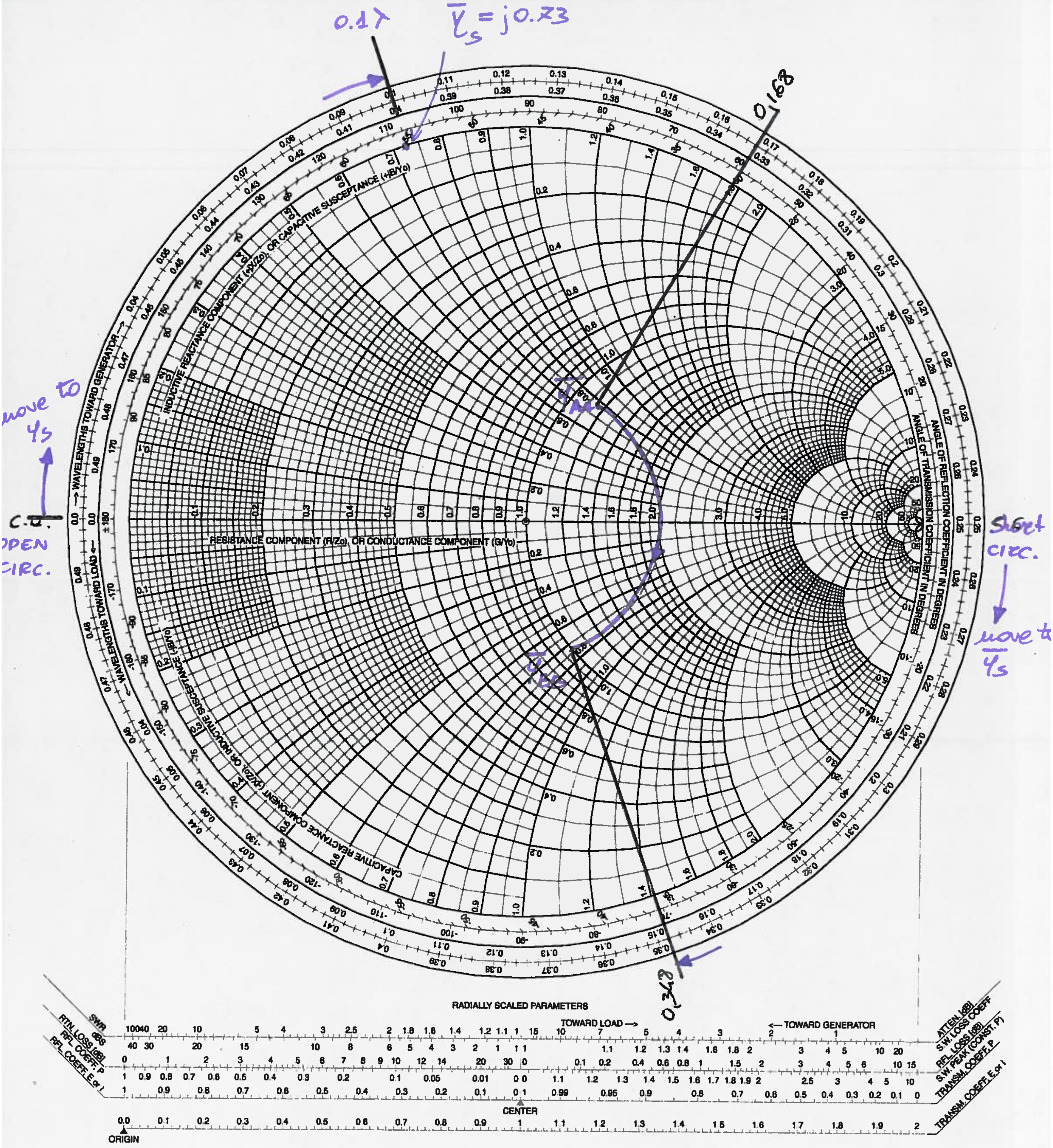
$d_s = 0.35\,\lambda = 10.5$ cm



open circuit stub → 0,1 λ

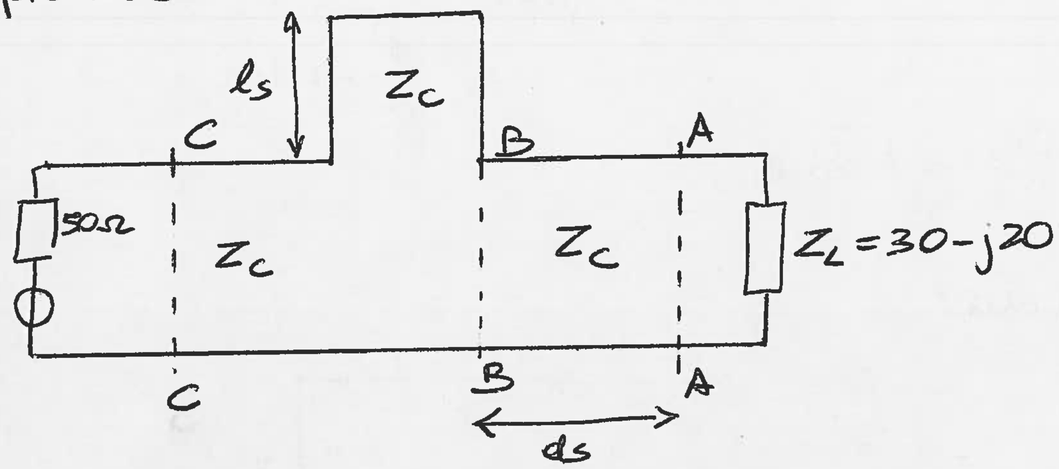
short circuit stub → 0,1 λ + 0,25 λ = 0.35 λ

Smith Chart



EXERCISE (Impedance matching with stub lines)

Calculate the length of the lines d_S and l_S in order to have perfect impedance matching at the generator section for the transmission line shown in the picture



$f = 1\,GHz$

$\lambda = 30\,cm$

The normalized impedance at section A is

$$\bar{z}_{AA} = \frac{Z_L}{Z_C} = 0{,}6 - j\,0.4$$

The line section d_S must move the point $\bar{z}_{AA}$ to a point with $Re\{\bar{z}_{BB}\} = 1$. Rotating $\bar{z}_{AA}$ around the center of the Smith Chart, the first solution is

$$\bar{z}_{BB} = 1 + 0.75\,j$$

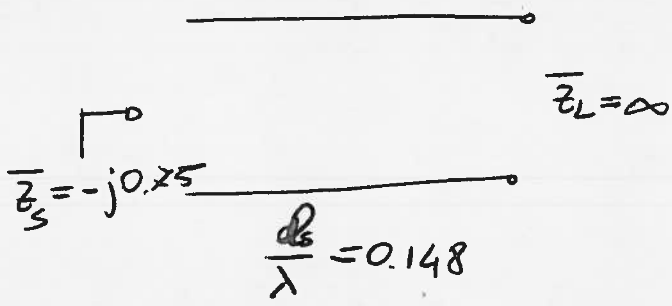
The length d_S is given by

$$\frac{d_S}{\lambda} = 0{,}154 + 0{,}082 = 0{,}236 \quad \Rightarrow d_S = 0{,}236\,\lambda = 7{,}08\,cm$$

The stub line must add an impedance $\bar{z}_s = -j0.75$ in order to cancel out the imaginary part of $\bar{z}_{BB}$

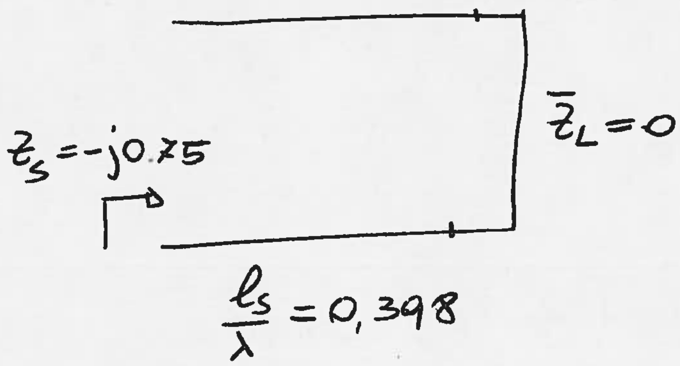
By using an <u>open circuit</u> stub

$\ell_s = 0.148\,\lambda = 4{,}44$ cm



By using a short circuit stub

$\ell_s = 398\,\lambda = 11{,}94$ cm



open circuit stub $\rightarrow$ $\ell_S = 0.148\lambda$

short circuit stub $\rightarrow$ $\ell_S = 0{,}398\lambda$

Smith Chart