A note on Leaky wave antennas (G. G. Gentili)

Plane waves in a lossless medium are represented by $e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}$ where $\boldsymbol{k}$ can be real or complex.

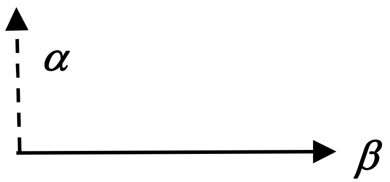
- When $\boldsymbol{k}$ is real we have a **uniform** wave, $|\boldsymbol{k}| = k = \omega\sqrt{\mu\epsilon}$

- When $\boldsymbol{k}$ is complex we have an evanescent wave, $\boldsymbol{k} = \boldsymbol{\beta} - j\boldsymbol{\alpha}$, with $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ real vectors. $\boldsymbol{\alpha}$ points towards the attenuation direction and $\boldsymbol{\beta}$ towards the propagation direction. In a lossless medium we must have

$$\beta^2 - \alpha^2 = k^2 = \omega^2\mu\epsilon$$

$$\boldsymbol{\beta} \perp \boldsymbol{\alpha}$$

So that, e.g.

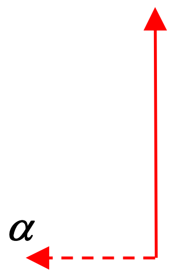


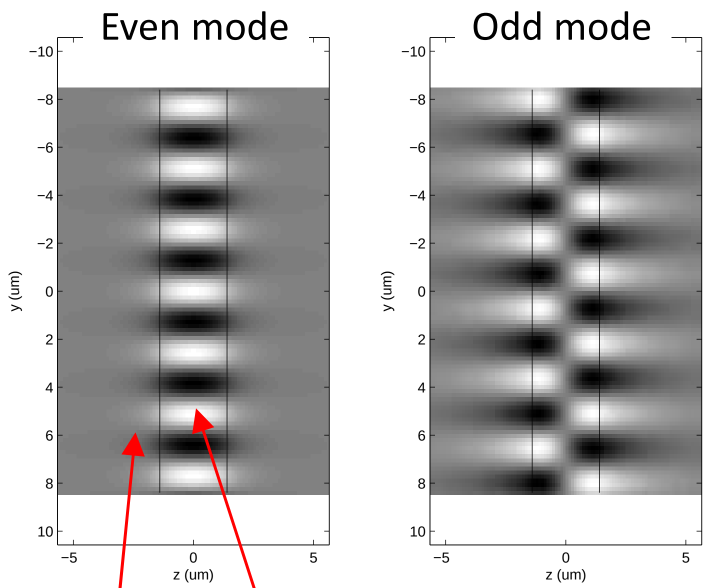
Must be $\beta > k$

Since $v_f = \frac{\omega}{\beta}$ and $v = \frac{\omega}{k}$ we have

$v_f < v$ So the evanescent wave is called **slow**

A typical slow wave is a guided mode (bound mode) in a dielectric waveguide





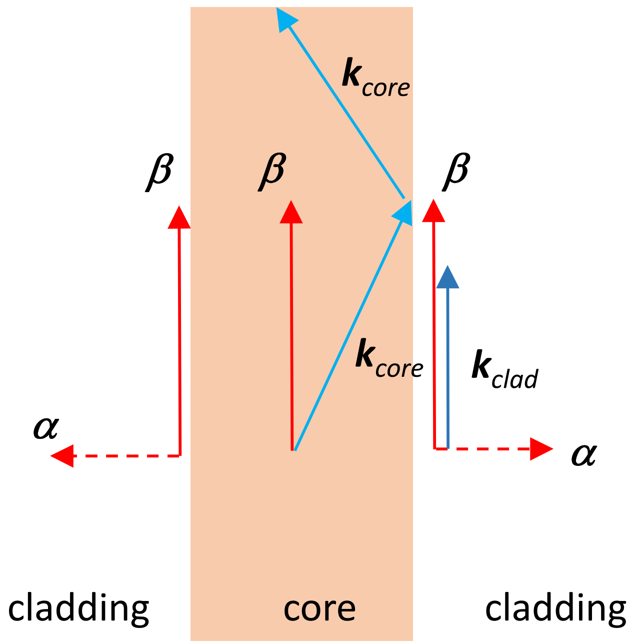
evanescent wave

Propagating wave bouncing at the side walls

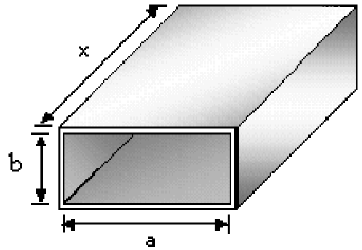
β is the longitudinal wavenumber and it is the same everywhere

In the outer medium (cladding), $\beta > k_{clad}$ (slow wave)

In the inner medium (core), $\beta < k_{core}$ (fast wave w.r.t. the inner medium)

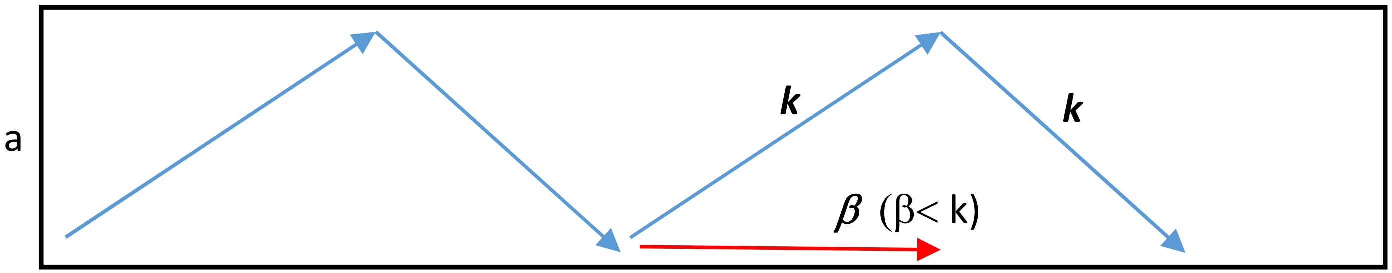


A leaky wave is a **fast** wave, with $\beta < k$ So $v_f > v$

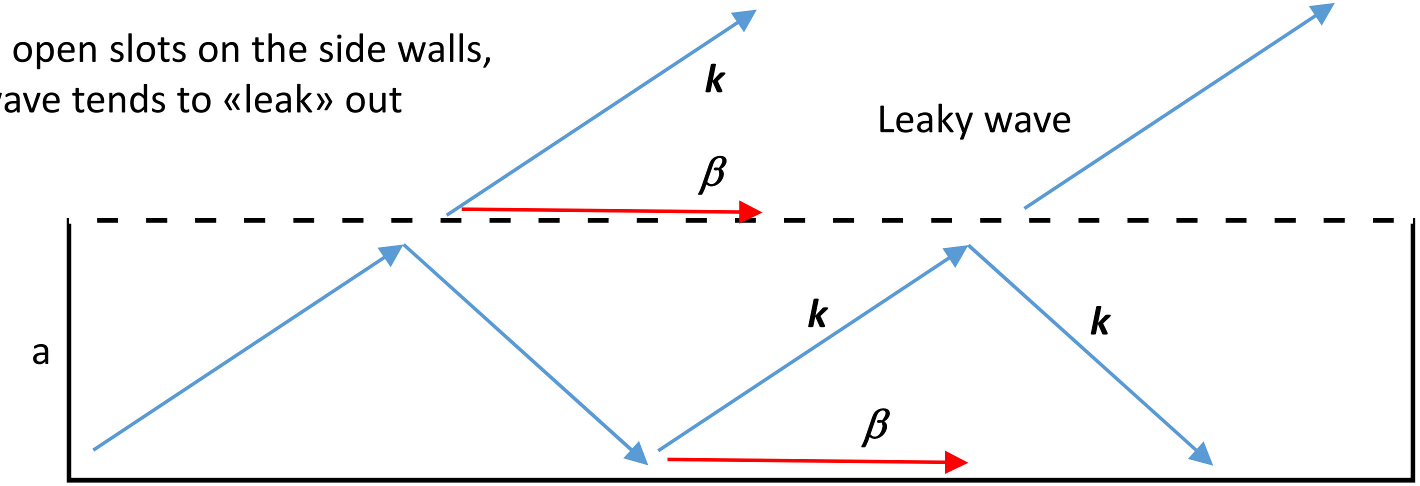


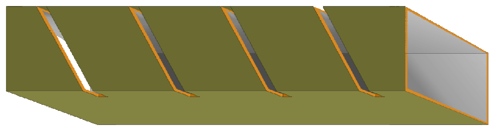
You normally have fast waves in metallic waveguides (propagating modes are fast)

You can imagine bouncing waves on the walls



If you open slots on the side walls, the wave tends to «leak» out



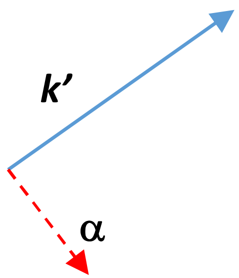


The leaky wave has some curious features. In the figure, $\beta = k_z$ and $k_x = \sqrt{k^2 - \beta^2}$ is **real** (positive radicand)

A real k_x is required to have radiation (and therefore a fast wave must be propagating, with $\beta = k_z < k$) !

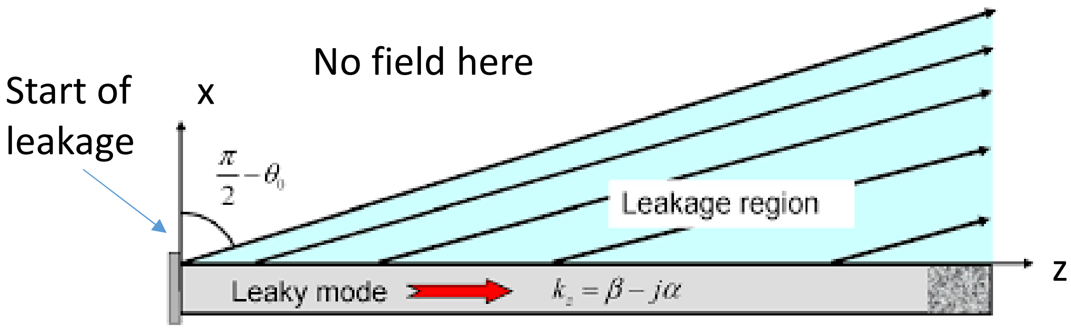
But since we have radiation we have attenuation along z. Therefore a (small) component of α comes into play. What is the consequence on the air wave? It has some alfa, therefore **k** must increase (e.g. it becomes **k'**), directed approximately like **k** and this allows us to find the other component of α, namely α_x. The condition is $\boldsymbol{\alpha} \perp \boldsymbol{k}'$

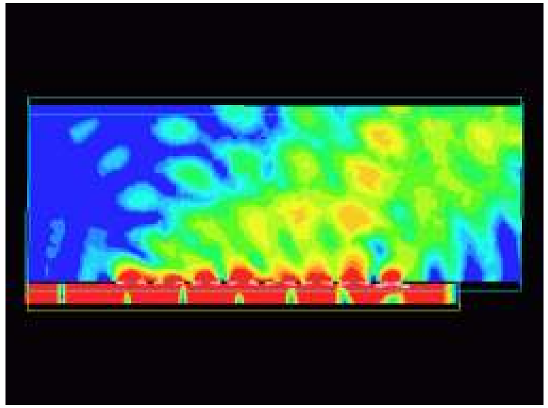
and the only possibility is

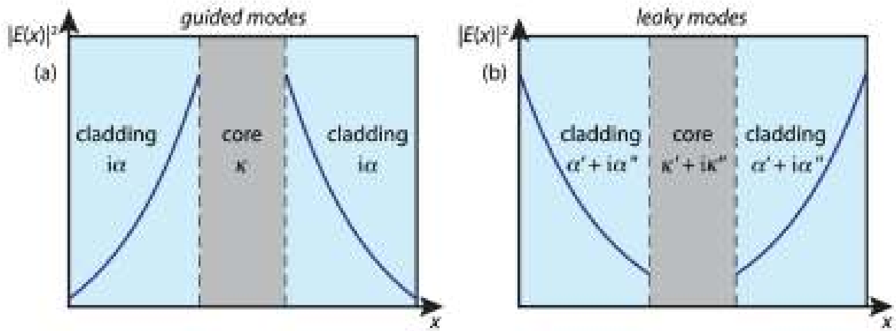


Because a positive z-component of α must be present (radiation loss)

So we have **attenuation** along z and **amplification** along x. But this only holds when leakage starts (as in the picture)

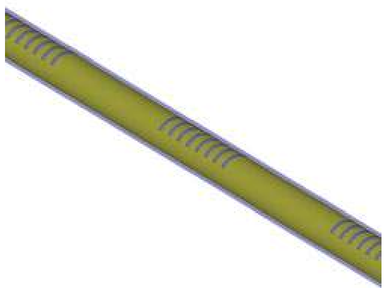


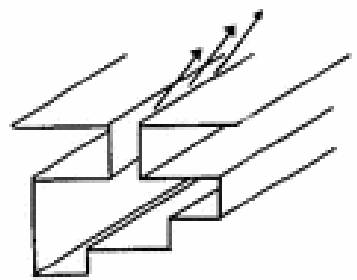




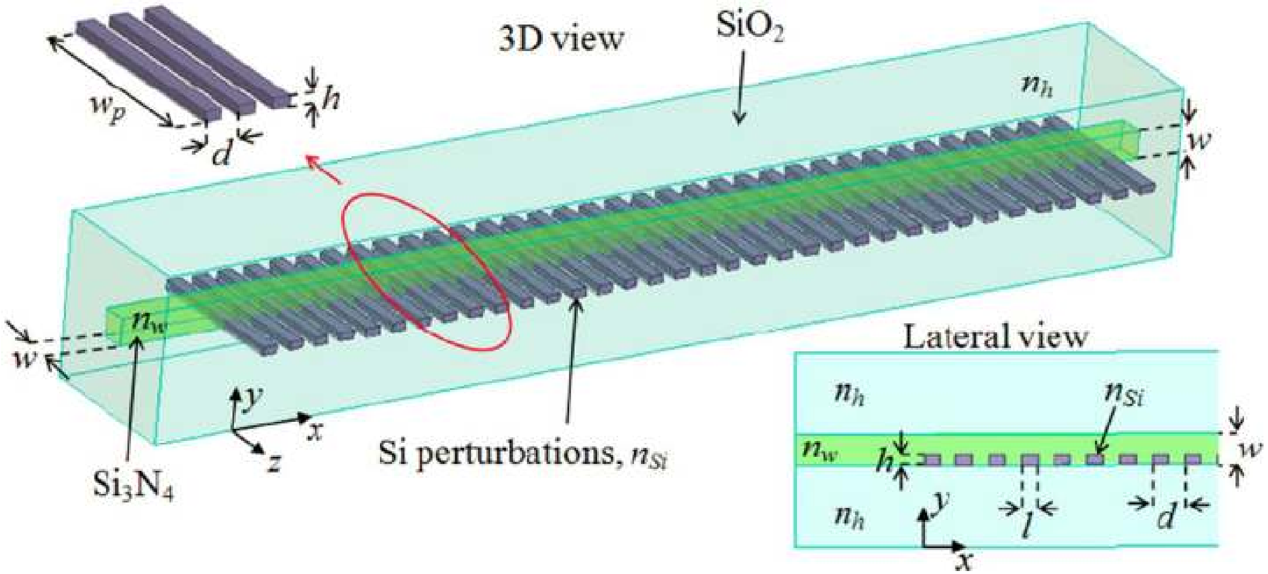
A comparison between guided mode and leaky mode (or leaky wave). Notice the increase of |E| away from the center

Other typical examples of uniform leaky wave antennas:
Leaky cable and stepped waveguide





Another possibility to get a fast wave is offered by **periodic structures**



Adding a periodic pattern (period d) produces a set of space harmonics with $k_x = \beta + n\frac{2\pi}{d}$ with n=0, ±1, ±2, …

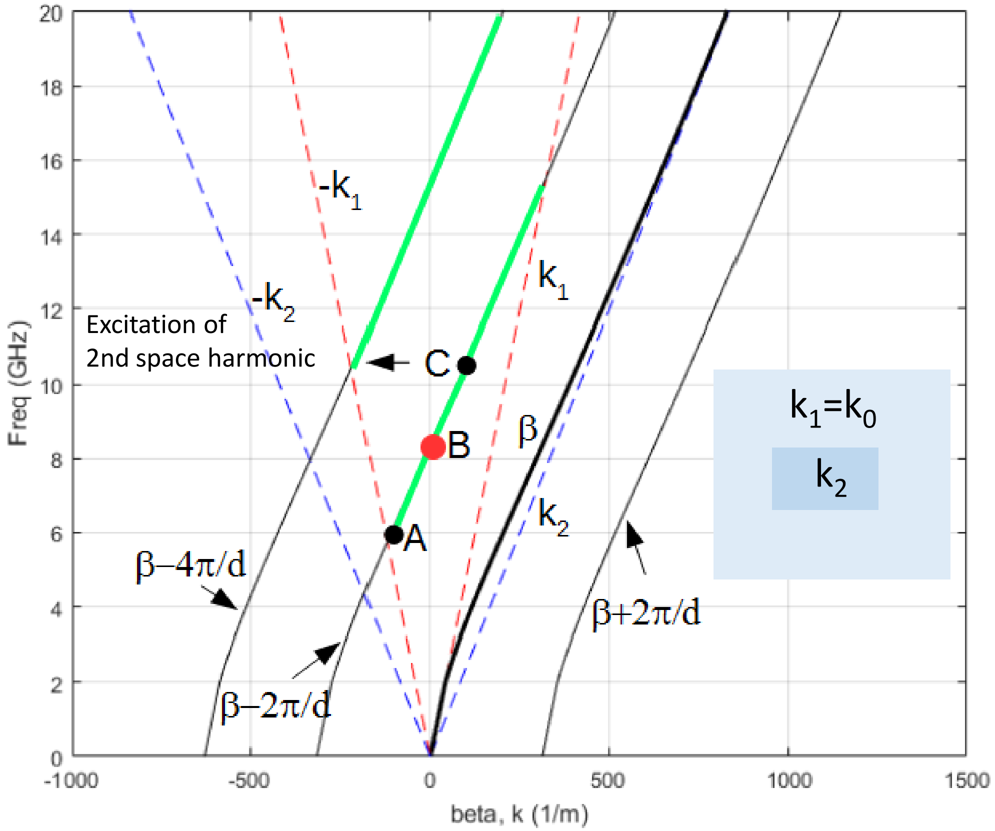
where β is the propagation constant of the guided mode with no pattern (a slow wave).

The wave dependence along x is therefore $\exp\left(-j\beta x - jn\frac{2\pi}{d}x\right)$. But then since $k_x^2 + k_y^2 = k^2$, along y we have

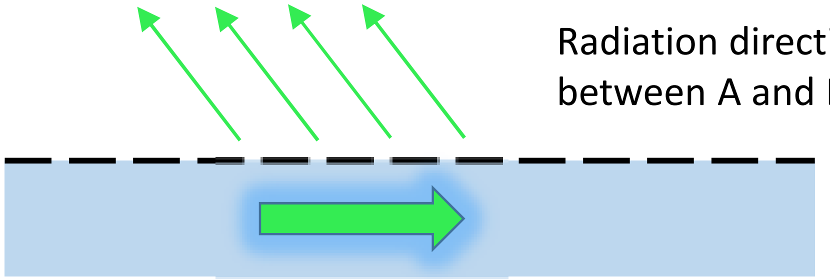
$$k_y = \sqrt{k^2 - (\beta + n\frac{2\pi}{d})^2}$$

A fast wave is obtained if $k_y = \sqrt{k^2 - (\beta + n\frac{2\pi}{d})^2}$ is positive real, i.e. $\beta + n\frac{2\pi}{d}$ must be **less** than k in air, as expected

Since β>k (the guided mode is slow in air), *n* must be **negative**. Starting from the dispersion curve of the mode in the dielectric waveguide, we can add the various space harmonics $n\frac{2\pi}{d}$

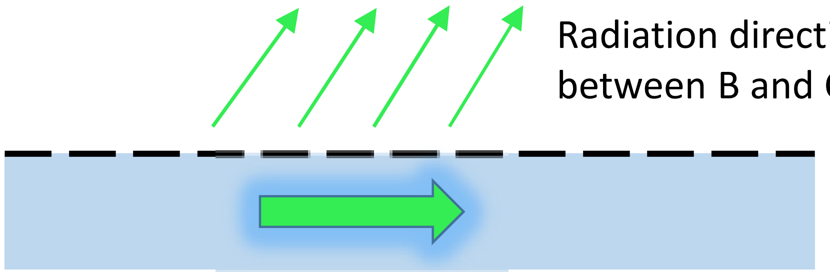


In the figure, the black heavy line is the dispersion curve of the guided mode (no periodic pattern). It lies between the red and blue k-curves (for the two media) In order to get radiation we must fall inside the red cone (k_x less than k_1 where k_1 is air, k_2 is the dielectric medium). This condition is labelled in green color.
You only want a **single** space harmonic to radiate, so the only allowed regions are between point A and B (backward radiation) and between point B and C (forward radiation). Point B, for which $\beta - \frac{2\pi}{d} = 0$ and neighborhoods are not used because we have too strong attenuation and mismatch.
(note that the axis units depend on the size of the waveguide).



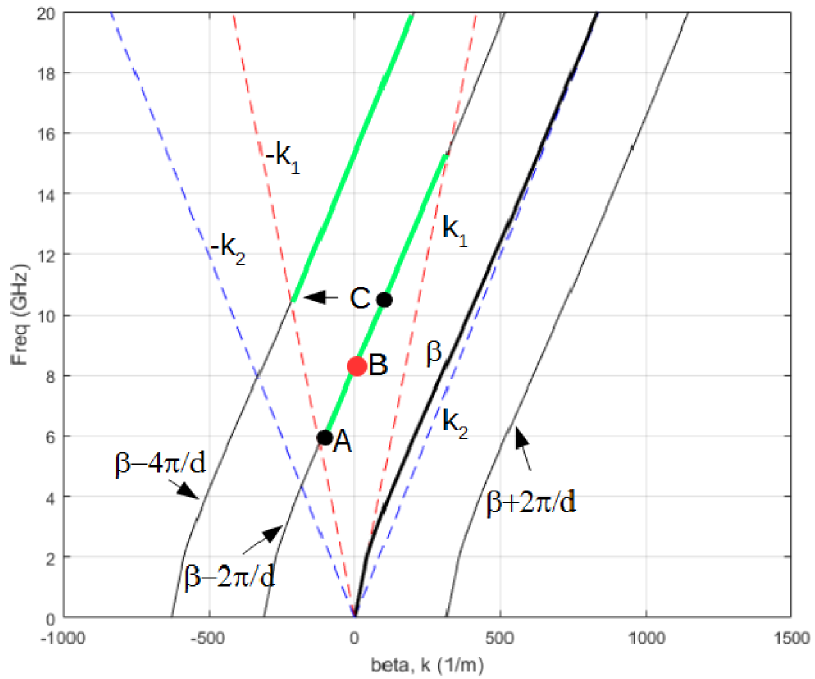
Radiation direction between A and B

Mode propagation $\beta - \dfrac{2\pi}{d}$



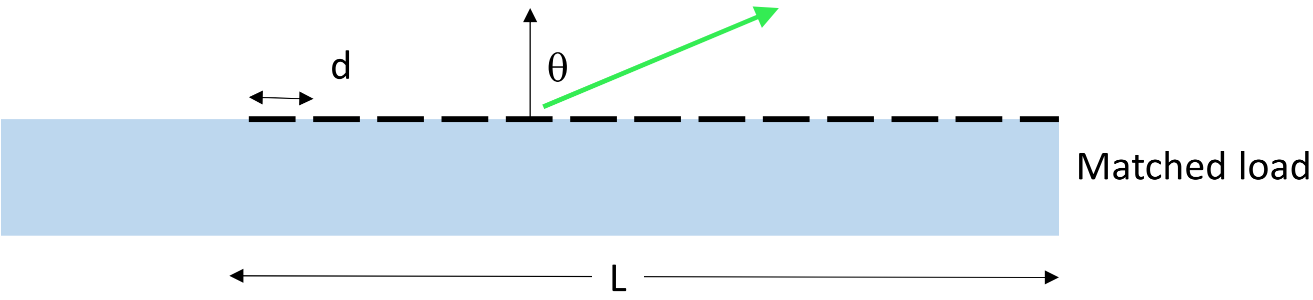
Radiation direction between B and C

Mode propagation $\beta - \dfrac{2\pi}{d}$



You have for free a scanning capability with frequency. In periodic structures this also comprises the backward direction. In fast modes in uniform structures you only can scan forward.

Design hints: period d and length L



Attenuation α due to radiation should not be too large, otherwise the effective aperture gets too small.
But α also affects efficiency (too small α means that most of the power gets absorbed by the matched load).
Beamwidth B_W depends on L as usual.

$$B_w \cong \frac{1}{\frac{L}{\lambda_0} cos\theta}$$

Finally, in order to reduce sidelobe level, tapering can be used