EX 1

A PLANE WAVE PROPAGATES THROUGH SEA WATER ($\sigma = 4\ S/m$, $\varepsilon_r = 81$, $\mu_r = 1$) IN THE $\vec{u}_z$ DIRECTION.

THE ELECTRIC FIELD IS GIVEN BY:

$$\vec{\mathcal{E}}(z,t) = E_0\, e^{-\alpha z} \cdot \cos\left(6\pi \cdot 10^3 t - \beta z\right) \vec{u}_x \quad [V/m]$$

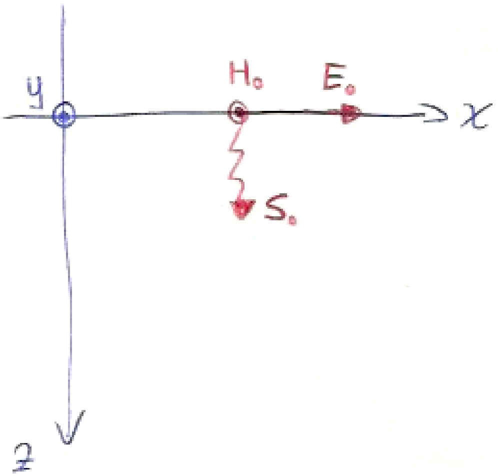
WHERE E_0 IS THE FIELD AMPLITUDE JUST BELOW THE SURFACE ($z = 0$).

ⓐ CALCULATE γ, α, β

ⓑ CALCULATE λ, v_f, δ, η

ⓒ EVALUATE THE EXPRESSION OF $\vec{\mathcal{H}}(z,t)$

ⓓ A SUBMARINE AT A DEPTH OF 100 m HAS AN ANTENNA CAPABLE TO RECEIVE SIGNALS WITH ELECTRIC FIELD AMPLITUDE AS LOW AS $1\,\mu V/m$.
CALCULATE THE MINIMUM FIELD $|E_{0,min}|$ AT THE SURFACE TO COMMUNICATE WITH THE SUBMARINE (AND ALSO THE VALUE OF $|H_{0,min}|$)



$\sigma = 4\ S/m$

$\varepsilon_r = 81$

$\mu_r = 1$

• THE GENERAL EXPRESSION OF FIELD IS:

$$\vec{\mathcal{E}}(z,t) = E_0 \cdot e^{-\alpha \cdot z} \cdot \cos\left(\omega t - \beta z + \cancel{\phi_{E_0}}\right) \vec{\mu}_x$$

• $\omega = 2\pi f = 6\pi \cdot 10^3$

$$f = \frac{6\pi \cdot 10^3}{2\pi} = 3 \cdot 10^3 \ Hz = \boxed{3KHz}$$

• $\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

TO CALCULATE α AND β EASILY WE CAN EVACUATE IF THE MEDIUM BEHAVES AS A GOOD CONDUCTOR OR AS A GOOD DIELECTRIC:

$$\tan\phi = \frac{\sigma}{\varepsilon_0 \cdot \varepsilon_r \cdot \omega} = \frac{4}{8{,}854 \cdot 10^{-12} \cdot 81 \cdot 6\pi \cdot 10^3} = 2{,}96 \cdot 10^5 \gg 1$$

(LOSS TANGENT)

THE SEA WATER AT THIS FREQUENCY (VERY LOW) BEHAVES AS A <u>GOOD CONDUCTOR</u>, SO:

$$\alpha = \beta = \sqrt{\frac{\omega \mu_0 \mu_r \sigma}{2}} = \sqrt{\frac{6\pi \cdot 10^3 \cdot 4\pi \cdot 10^{-7} \cdot 1 \cdot 4}{2}} = 0{,}218$$

$$\alpha = 0{,}218 \ \frac{Np}{m} \longrightarrow \delta = \frac{1}{\alpha} = 4{,}59 \ m$$

(SKIN EFFECT)

δ IS THE PENETRATION THICKNESS OF POWER INSIDE THE CONDUCTOR SURFACE.

$$\beta = 0{,}218 \ \frac{rad}{m}$$

$\boxed{b}$ • TO CALCULATE THE WAVELENGTH IN THE MEDIUM WE NEED THE PHASE CONSTANT β:

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0{,}218} = \boxed{28{,}8 \ m}$$

(IN THE VACUUM THE λ AT THIS LENGTH IS: $\lambda = \frac{c}{f} = 100 \ Km$ SO 3 ORDERS OF MAGNITUDE)

MLARLY, THE PHASE VELOCITY DEPENDS ON β:

$$v_f = \frac{\omega}{\beta} = \frac{6\pi \cdot 10^3 \; Hz}{0{,}218 \; \frac{rad}{m}} = 8{,}65 \cdot 10^4 \; m/s$$

(IN THE VACUUM $v_f = c = 3 \cdot 10^8$, 3 ORDERS OF MAGNITUDE GREATER)

- THE INTRINSIC IMPEDENCE FOR GOOD CONDUCTORS IS:

$$\eta = (1+j)\sqrt{\frac{\omega \mu_0 \mu_r}{2\sigma}} = (1+j)\sqrt{\frac{6\pi \cdot 10^3 \cdot 4\pi \cdot 10^{-7} \cdot 1}{2 \cdot 4}} =$$

$$= 5{,}44 \cdot 10^{-2}(1+j) \quad [\Omega]$$

(IN THE VACUUM IS $\eta_0 = 377\,\Omega$, 3 ORDERS OF MAGNITUDE GREATER)

$\boxed{C}$ WE KNOW THAT ADOPTING THE PHASOR NOTATION, $\vec{E}$ AND $\vec{H}$ ARE CONNECTED TO EACH OTHER BY MEANS OF η:

$$\frac{\vec{E}}{\vec{H}} = \eta \longrightarrow \vec{H} = \frac{\vec{E}}{\eta} = \frac{\vec{E}}{|\eta|} e^{-j\varphi_\eta}$$

$$\vec{E} = E_0 e^{-\gamma z}$$

$$\eta = |\eta| \cdot e^{j\varphi_\eta} \Longrightarrow |\eta| = \sqrt{(5{,}44 \cdot 10^{-2})^2 + (5{,}44 \cdot 10^{-2})^2} = 7{,}7 \cdot 10^{-2}$$

$$\varphi_\eta = \arctan\left(\cancelto{1}{\frac{Im[\eta]}{Re[\eta]}}\right) = \frac{\pi}{4}$$

$$\vec{H} = \frac{E_0 e^{-\gamma z}}{|\eta|} e^{-j\varphi_\eta} = \frac{E_0}{|\eta|} e^{-\kappa z} \cdot e^{-j\beta z} \cdot e^{-j\varphi_\eta}$$

$$\vec{\mathcal{H}}(z,t) = Re\left[\vec{H} \cdot e^{j\omega t}\right] \vec{u}_y = Re\left[\frac{E_0}{|\eta|} e^{-\kappa z} \cdot e^{-j\varphi_\eta} \cdot e^{-j\beta z} e^{j\omega t}\right] \vec{u}_y =$$

$$= \frac{E_0}{|\eta|} \cdot e^{-\alpha z} \cdot \cos(\omega t - \beta z - \varphi_\eta)\, \vec{u}_y =$$

$$= \frac{E_0}{7{,}7 \cdot 10^{-2}} \cdot e^{-0{,}218\, z} \cdot \cos\left(6\pi \cdot 10^3 t - 0{,}218 \cdot z - \frac{\pi}{4}\right) \vec{u}_y$$

$$= 13 \cdot E_0 \cdot e^{-0{,}218 \cdot z} \cdot \cos\left(6\pi \cdot 10^3 t - 0{,}218\, z - \frac{\pi}{4}\right) \vec{u}_y \quad \left[\frac{A}{m}\right]$$

INVERSE OF THE APLITUDE OF INTRINSIC IMPEDENCE

PHASE OF INTRINSIC IMPEDENCE

$\vec{H}$ IS PERPENDICULAR TO $\vec{E}$

d A depth of 100 m MEANS $z = 100$ m:

$$|\vec{E}| = E_0 \cdot e^{-\alpha z} = E_0 \cdot e^{-0{,}218 \cdot 100} = \boxed{E_0 \cdot 3{,}4 \cdot 10^{-10} \geq 10^{-6} \frac{V}{m}}$$

$$|E_{0,min}| \geq \frac{10^{-6}}{3{,}4 \cdot 10^{-10}} = \boxed{2{,}94 \cdot 10^3 \frac{V}{m}}$$

$$|H_{0,min}| = \frac{|E_{0,min}|}{|\eta|} = \frac{2{,}94 \cdot 10^3\, V/m}{7{,}7 \cdot 10^{-2}\, \Omega} = \boxed{38{,}1 \cdot 10^3 \frac{A}{m}}$$

EX 2

AT A FREQUENCY OF 10 MHz SEA WATER HAS $\varepsilon_r = 81$, $\sigma = 4$ S/m

WHILE LAKE WATER HAS $\varepsilon_r = 81$, $\sigma = 4 \cdot 10^{-3}$ S/m

WHICH MEDIUM IS THE MOST SUITABLE FOR RADIO COMMUNICATION?

THE GOODNESS OF WAVE PROPAGATION IS RELATED TO THE PENERATION LENGTH (δ) THROUGH THE MEDIUM, WHICH IS DEPENDENT ON THE ATTENUATION CONSTANT (α).

THE LOSS TANGENT IS:

$$\tan\phi = \frac{\sigma}{\omega \varepsilon_r \cdot \varepsilon_0} \rightarrow \frac{4}{2\pi \cdot 10^7 \cdot 81 \cdot 8{,}854 \cdot 10^{-12}} = 88{,}7$$

GOOD CONDUCTOR

$$\tan\phi = \frac{\sigma}{\omega \varepsilon_r \cdot \varepsilon_0} \rightarrow \frac{4 \cdot 10^{-3}}{2\pi \cdot 10^7 \cdot 81 \cdot 8{,}854 \cdot 10^{-12}} = 0{,}08$$

GOOD DIELECTRIC

Sea Water (GOOD CONDUCTOR)

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} \Longrightarrow \delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{2\pi \cdot 10^7 \cdot 4\pi \cdot 10^{-7} \cdot 4}} =$$

$$= \boxed{8 \text{ cm}}$$

Lake Water (GOOD DIELECTRIC)

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \Longrightarrow \delta = \frac{1}{\alpha} = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}} = \frac{2}{4 \cdot 10^{-3}} \cdot \sqrt{81} \cdot \frac{1}{377} =$$

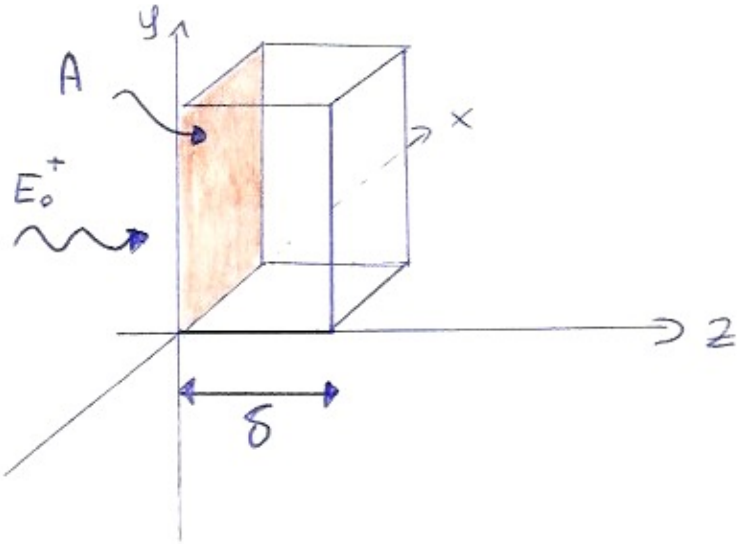
$$= \boxed{11{,}9 \text{ m}}$$

NOTICE THAT FOR GOOD DIELECTRIC α DOESN'T DEPEND ON THE FREQUENCY

SO WE SHOULD PREFER PROPAGATING INTO LAKE WATER

EX 3

A UNIFORM PLANE WAVE WITH AMPLITUDE $|\vec{E}| = 100$ V/m AND FREQUENCY $f = 7$ GHz PROPAGATES THROUGH SEA WATER ($\sigma = 4$ S/m, $\varepsilon_r = 81$). CALCULATE THE POWER DISSIPATED IN A WATER BLOCK WITH A SURFACE OF 10 cm^2 AND A THIKKNESS EQUAL TO THE PENETRATION DEPTH δ.



$$\varepsilon_0 = 8{,}854 \cdot 10^{-12} \text{ F/m}$$

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ H/m}$$

$$A = 10 \text{ cm}^2 = 10^{-3} \text{ m}^2$$

THE <u>POWER DENSITY</u> OF THE WAVE (NEGLECTING THE PRESENCE OF COUNTER-PROPAGATING / BACKWARD COMPONENT) IS GIVEN BY:

$$S^+ = \frac{1}{2} \frac{|E_0^+|^2}{|\eta|} \cdot e^{-2\alpha z} \cdot \cos(\varphi_\eta) \quad \left[W/m^2 \right]$$

SO WE NEED TO CALCULATE

→ η : INTRINSIC IMPEDENCE OF THE MEDIUM

→ α : ATTENUATION CONSTANT (LOSSY MEDIUM)

THEN THE DISSIPATED POWER WILL BE SIMPLY CALCULATED MAKING THE DIFFERENCE BETWEEN THE WAVE POWER BEFORE THE BLOCK ($z = 0$) AND AFTER THE BLOCK ($z = \delta$)

i'S CHECK THE LOSS TANGENT:

$$\tan\phi = \frac{\sigma}{\omega\varepsilon} = \frac{4}{2\pi \cdot 7 \cdot 10^{9} \cdot 8{,}854 \cdot 10^{-12} \cdot 81} = 0{,}127$$

SINCE $\tan\phi$ IS NEITHER MUCH GREATER THAN 1 NOR MUCH LOWER THAN 1 THIS MEDIUM (AT THIS FREQUENCY) CAN'T BE APPROXIMATED AS A GOOD CONDUCTOR OR DIELECTRIC. SO:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j(2\pi \cdot 7 \cdot 10^{9})(4\pi \cdot 10^{-7})}{4 + j(2\pi \cdot 7 \cdot 10^{9}) \cdot (8{,}854 \cdot 10^{-12} \cdot 81)}} =$$

$$= \sqrt{\frac{j\,5{,}53 \cdot 10^{4}}{4 + j\,31{,}54}} = \sqrt{\frac{5{,}53 \cdot 10^{4} \cdot e^{j90^\circ}}{31{,}8 \cdot e^{j82{,}77^\circ}}} =$$

$$= \sqrt{1739 \cdot e^{j(7{,}23^\circ)}} = \sqrt{1739} \cdot e^{j\left(\frac{7{,}23^\circ}{2}\right)} =$$

$$= 41{,}7 \cdot e^{j(3{,}615^\circ)} \quad [\Omega]$$

$$\boxed{\begin{array}{l} |\eta| = 41{,}7\ \Omega \\ \varphi_\eta = 3{,}615^\circ \end{array}}$$

ÍNSIDE THE BLOCK THE POWER DENSITY IS:

$$S^+(z=0) = \frac{1}{2} \cdot \frac{|E_0^+|^2}{|\eta|} \cos(\varphi_\eta) \quad \left[\frac{W}{m^2}\right]$$

AFTER IT:

→ SO I DON'T NEED TO CALCULATE α

$$S^+(z=\delta) = \frac{1}{2} \cdot \frac{|E_0^+|^2}{|\eta|} \cdot e^{-2\overbrace{(\alpha\cdot\delta)}^{1}} \cdot \cos(\varphi_\eta) \quad \left[\frac{W}{m^2}\right]$$

SO:

$$P_{DISSIP} = P^+(z=0) - P^+(z=\delta) = A \cdot S^+(z=0) - A \cdot S^+(z=\delta) =$$

$$= A \cdot \left[S^+(z=0) - S^+(z=\delta)\right] =$$

$$= A \cdot \left[\frac{1}{2}\frac{|E_0^+|^2}{|\eta|} \cos\varphi_\eta - \frac{1}{2}\frac{|E_0^+|^2}{|\eta|} \cdot e^{-2} \cdot \cos\varphi_\eta\right] =$$

$$= \frac{A}{2} \cdot \frac{|E_0^+|^2}{|\eta|} \cdot \cos\varphi_\eta \cdot \left[1 - e^{-2}\right] =$$

$$= \frac{10^{-3}}{2} \cdot \frac{(100)^2}{41,7} \cdot \underbrace{\cos(3,615°)}_{0,998} \cdot \underbrace{\left[1 - \frac{1}{e^2}\right]}_{0,865} =$$

$$= \boxed{0,1035 \ W}$$

EX 4

A UNIFORM PLANE WAVE PROPAGATES THROUGH A MEDIUM WITH $\varepsilon_r = 36$; $\mu_r = 4$; $\sigma = 1$ S/m.

THE ELECTRIC FIELD IS GIVEN BY:

$$\mathcal{E}_z(x,t) = 100 \cdot e^{-\alpha x} \cdot \cos\left(10\pi \cdot 10^8 \cdot t - \beta x\right) \vec{u}_z \ \left[V/m\right]$$

- CALCULATE THE EXPRESSION OF ATTENUATION CONSTANT, PHASE CONSTANT AND FIND THE EXPRESSION OF $\vec{\mathcal{H}}$.

LET'S CHECK THROUGH THE LOSS TANGENT IF THE MEDIUM CAN BE APPROXIMATED:

$$\tan\phi = \frac{\sigma}{\omega\varepsilon} = \frac{1}{(10\pi \cdot 10^8)\cdot(8{,}854 \cdot 10^{-12} \cdot 36)} = 1$$

$$\omega = 10\pi \cdot 10^8 \longrightarrow f = \frac{\omega}{2\pi} = 5 \cdot 10^8 = \boxed{500 \text{ MHz}}$$

NO, THIS MEDIUM CAN'T BE APPROXIMATED!
SO WE NEED TO USE THE FULL EXPRESSION OF $\gamma = \alpha + j\beta$:

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon} =$$

$$= \sqrt{\left(j \cdot 10^9\pi \cdot 4\pi \cdot 10^{-7} \cdot 4\right) - \left(10^8 \cdot \pi^2 \cdot 4\pi \cdot 10^{-7} \cdot 4 \cdot 8{,}854 \cdot 10^{-12} \cdot 36\right)} =$$

$$= \sqrt{\underbrace{15\,791\, j - 15\,813}_{K}} = \sqrt{2{,}23 \cdot 10^4} \cdot e^{j\left(\frac{135{,}04°}{2}\right)} =$$

$$|K| = \sqrt{(15791)^2 + (15813)^2} = 2{,}23 \cdot 10^4$$

$$\measuredangle K = 90° + \arctan\left(\frac{15813}{\ldots}\right) = 135{,}04°$$

$$= 149,3 \cdot e^{j(67,52°)} = 149,3 \cdot \cos(67,52°) + j\,149,3 \cdot \sin(67,52°) =$$

$$= \boxed{57 + 138\,j} = \alpha + j\beta$$

$$\boxed{\begin{aligned} \alpha &= 57 \text{ Np/m} \\ \beta &= 138 \text{ rad/m} \end{aligned}}$$

NOW, IF I WANT TO GET $\vec{H}$ I NEED TO FIND THE INTRINSIC WAVE IMPEDENCE η:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)} \cdot \left(\frac{\sigma - j\omega\varepsilon}{\sigma - j\omega\varepsilon}\right)} = \sqrt{\frac{\omega^2\mu\varepsilon + j\omega\mu\sigma}{\sigma^2 + \omega^2\varepsilon^2}} =$$

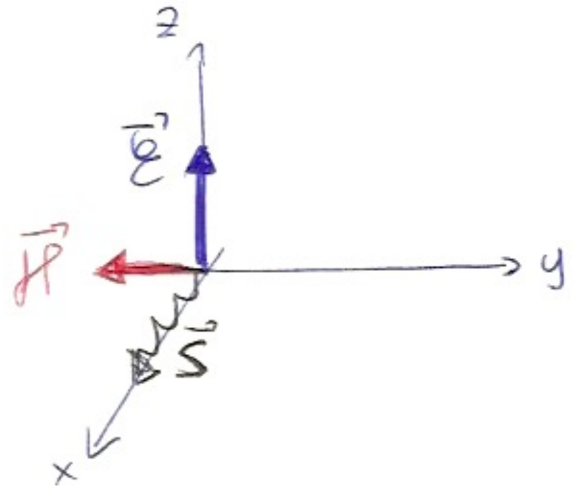
$$= \sqrt{7896 + j\,7888} = 97,6 + j\,40,4 \ [\Omega]$$

$$|\eta| = \sqrt{97,6^2 + 40,4^2} = \sqrt{9525,7 + 1632,2} =$$

$$= 105,63$$

$$\varphi_\eta = \arctan\left(\frac{40,4}{97,6}\right) = 22,49° \rightarrow 0,39 \text{ rad}$$

ÒW I HAVE ALL THE ELEMENTS TO DETERMINE $\vec{H}$:



- THE ORIENTATION OF THE MAGNETIC FIELD $\vec{H}$ DEPENDS ON THE FACT THAT:

→ THE WAVE PROPAGATES IN $\vec{u}_x$ DIRECTION

→ THE ELECTRIC FIELD IS ORIENTED AS $\vec{u}_z$

↓

SO $\vec{H}$ IS DIRECTED AS $-\vec{u}_y$

$$\frac{\vec{E}}{\vec{H}} = \eta \rightarrow \boxed{\vec{H} = \frac{\vec{E}}{\eta}} = \left[\frac{E_0 \cdot e^{-\alpha x} \cdot e^{-j\beta x}}{|\eta| \cdot e^{j\varphi_\eta}}\right] (-\vec{u}_y)$$

$$\vec{H}_y(x,t) = \mathrm{Re}\left[\vec{H} \cdot e^{j\omega t}\right] =$$

$$= \frac{E_0 \cdot e^{-\alpha x}}{|\eta|} \cdot \cos(\omega t - \beta x - \varphi_\eta)\,(-\vec{u}_y) =$$

$$= -\frac{100 \cdot e^{-57 \cdot x}}{105,63} \cdot \cos(\omega t - 138 \cdot x - 0,39)\,(\vec{u}_y)\left[\frac{A}{m}\right]$$

i̇ I WOULD HAVE USED THE APPROXIMATIONS....

→ GOOD CONDUCTOR :

$$\alpha = \beta \simeq \sqrt{\frac{\omega \mu \sigma}{2}} = 88{,}86 \, [m^{-1}]$$

→ GOOD DIELECTRIC

$$\alpha \simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = 62{,}8 \, [m^{-1}]$$

$$\beta \simeq \omega \sqrt{\mu \varepsilon} = 125{,}7 \, [rad/m]$$

SINCE WE FOUND $\alpha = 57 \, [m^{-1}]$ AND $\beta = 138 \, [rad/m]$ THE APPROXIMATION THAT PROVIDES THE "BEST" DESCRIPTION OF THE MEDIUM IS THE "GOOD - DIELECTRIC" ONE.

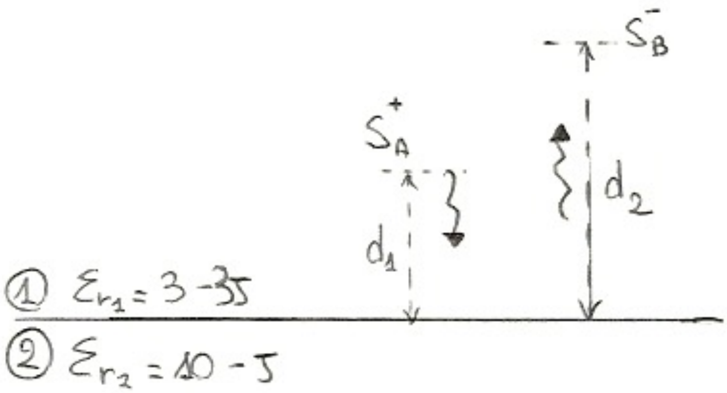
EX ~~1~~

A PLANE WAVE WITH A FREQUENCY $f = 10$ MHz AND POWER DENSITY $S_A^+ = 100 \ W/m^2$ AT A DISTANCE $d_1 = 1$ m FROM A DISCONTINUITY SURFACE PROPAGATES THROUGH A DIELECTRIC MEDIUM WITH $\varepsilon_{r_1} = 3 - 3j$.

CALCULATE:

ⓐ PHASE VELOCITY v_f

ⓑ POWER DENSITY OF THE BACK-REFLECTED WAVE AT A DISTANCE $d_2 = 2$ m FROM THE DISCONTINUITY WHEN THE DIELECTRIC CONSTANT OF THE MEDIUM ② IS $\varepsilon_{r_2} = 10 - j$.



ⓐ THE PHASE VELOCITY OF A WAVE IS DEFINED AS:

$$v_f = \frac{\omega}{\beta}$$

THE CONSTANT β (PHASE CONSTANT) CAN BE DERIVED FROM THE COMPLEX PROP. CONSTANT γ:

$$\gamma = \sqrt{j\omega\mu(\underset{0}{\cancel{\sigma}} + j\omega\varepsilon)} = \sqrt{(j\omega)^2 \mu_0 \varepsilon_0 \varepsilon_{r_1}} = j\omega\sqrt{\mu_0 \varepsilon_0 \varepsilon_{r_1}} = \alpha + j\beta$$

$$= j 2\pi \cdot 10^7 \cdot \sqrt{4\pi \cdot 10^{-7} \cdot 8,854 \cdot 10^{-12} (3 - 3j)} = e^{j90^\circ} \cdot 0,209 \cdot \sqrt{\sqrt{18} \cdot e^{-j45^\circ}} =$$

$$= 0,432 \cdot e^{j67,5^\circ} = 0,432 \cdot \cos(67,5^\circ) + j\, 0,432 \cdot \sin(67,5^\circ) =$$

$$= 0,165 + j\, 0,399 = \alpha + j\beta \quad [m^{-1}]$$

$$v_f = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \cdot 10^7}{0,399} = \boxed{1,57 \cdot 10^8 \ m/s}$$

(b) TO CALCULATE THE BACKSCATTERED POWER WE NEED TO CALCULATE THE REFLECTION COEFFICIENT Γ AT THE DISCONTINUITY BETWEEN MEDIUM (1) AND (2).

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\eta_1 = \sqrt{\frac{j\omega\mu}{\cancel{\sigma} + j\omega\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_{r1}}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sqrt{\frac{1}{\varepsilon_{r1}}} = 377 \cdot \sqrt{\frac{1}{4,24 \cdot e^{-j45°}}} =$$

$$\left(\varepsilon_{r1} = 3 - 3j = 4,24 \cdot e^{-j45°}\right)$$

$$= 377 \cdot \sqrt{0,236 \cdot e^{j45°}} = 169 + j70 \ [\Omega]$$

$$\eta_2 = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \sqrt{\frac{1}{\varepsilon_{r2}}} = 377 \cdot \sqrt{\frac{1}{10,05 \cdot e^{-j5,7°}}} = 119 + j5,9 \ [\Omega]$$

$$\Gamma = \frac{(119 + j5,9) - (169 + j70)}{(119 + j5,9) + (169 + j70)} = -0,218 - j0,165 = 0,274 \cdot e^{-j2,49°}$$

THE POWER DENSITY BACKREFLECTED DISTANCE d_2 IS:

$$S_B^- = \underbrace{S_A^+ \cdot e^{-2\alpha_1 d_1}}_{\text{POWER AT THE INTERFACE}} \cdot |\Gamma|^2 \cdot e^{-2\alpha_1 d_2} =$$

POWER BACKREFLECTED AT INTERFACE

POWER BACK[illegible] DISTANCE d_2

$$= 100\ \frac{W}{m^2} \cdot e^{-2 \cdot 0,165 \cdot 1} \cdot (0,274)^2 \cdot e^{-2 \cdot 0,165 \cdot 2} =$$

$$= 100 \cdot 0,719 \cdot 0,075 \cdot 0,5169 = \boxed{2,79\ W}$$

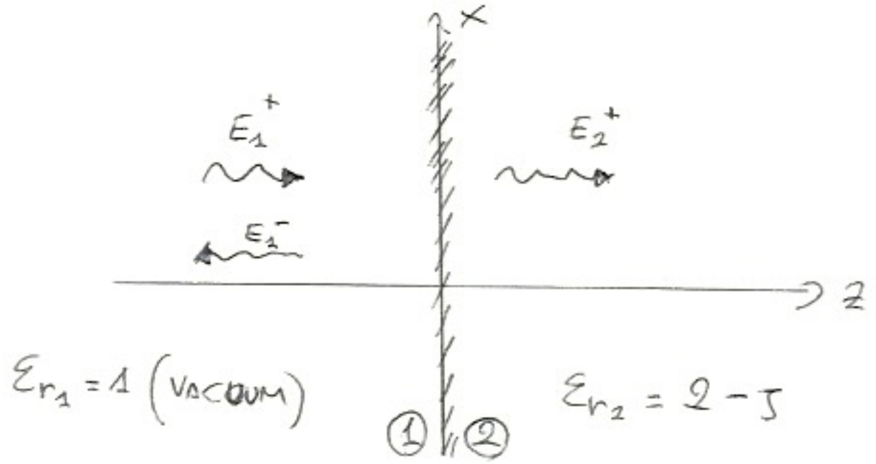
EX 2

A PLANE WAVE AT FREQUENCY $f = 1$ GHz IMPINGES ORTHOGONALLY ONTO THE SURFACE BETWEEN TWO DIELECTRICS AND CARRIES A POWER DENSITY $S_1^+ = 10 \frac{W}{m}$.

CALCULATE:

ⓐ THE TOTAL ELECTRIC FIELD E_{TOT} AT THE INTERFACE $(x = 0, z = 0)$

ⓑ THE TOTAL ELECTRIC FIELD E_2 IN THE SECOND MEDIUM AT DISTANCE $z = 5$ m FROM THE INTERFACE.



ⓐ FROM THE POWER DENSITY S_1^+ WE CAN CALCULATE THE INCIDENT ELECTRIC FIELD E_1^+ AT THE INTERFACE:

$$S_1^+ = \frac{1}{2} \frac{|E_1^+|^2}{\eta_1} \longrightarrow E_1^+ = \left(2 \cdot S_1^+ \cdot \eta_1\right)^{1/2} = \sqrt{2 \cdot 377 \cdot 10} =$$

$$= 86,83 \frac{V}{m}$$

$$(\eta_1 = \eta_0 = 377 \, \Omega)$$

THE TOTAL FIELD IN MEDIUM 1 AT THE INTERFACE IS:

$$E_{TOT} = E_1^+ + E_1^- = E_1^+ + \Gamma \cdot E_1^+ = (1 + \Gamma) \cdot E_1^+$$

$$E_1^- = \Gamma \cdot E_1^+ = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) \cdot E_1^+$$

$$\eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \sqrt{\frac{1}{\varepsilon_{r_2}}} = 377 \cdot \sqrt{\frac{1}{2{,}24 \cdot e^{-j26{,}56^\circ}}} =$$

$$= 377 \cdot 0{,}668 \cdot e^{+j13{,}28^\circ} = 245 + j\,57{,}85 \quad [\Omega]$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{245 + j\,57{,}85 - 377}{245 + j\,57{,}85 + 377} = -0{,}201 + j\,0{,}112$$

$$E_{TOT} = E_1^+(1+\Gamma) = 86{,}83 \cdot \left(1 - 0{,}201 + j\,0{,}112\right) =$$

$$= \boxed{69{,}35 + j\,9{,}7} \quad \left[\frac{V}{m}\right]$$

(b) THE TOTAL ELECTRIC FIELD IN THE SECOND MEDIUM IS:

$$E_2^+(z) = \underbrace{E_2^+(0)} \cdot e^{-\gamma_2 \cdot z} = E_{TOT} \cdot e^{-\gamma_2 \cdot z}$$

$$\frac{E_2^+}{E_1^+} = T = 1 + \Gamma$$

$$E_2^+ = E_1^+(1+\Gamma) = E_{TOT}$$ (THE FIELD IS TANGENT TO THE INTERFACE SO IS CONSERVATED IN THE TWO MEDIA)

$$\gamma_2 = j\omega\sqrt{\mu\varepsilon_2} = j\omega\sqrt{\mu_0 \varepsilon_0 \varepsilon_{r_2}} = 7{,}2 + j\,30{,}5 \quad [\Omega]$$

$$E_2^+(z = 5\,m) = \left(69{,}35 + j\,9{,}7\right) \cdot e^{-(7{,}2 + j\,30{,}5) \cdot 5} =$$

$$= 70{,}03\, e^{j7{,}96^\circ} \cdot e^{-36} \cdot e^{-j97^\circ} =$$

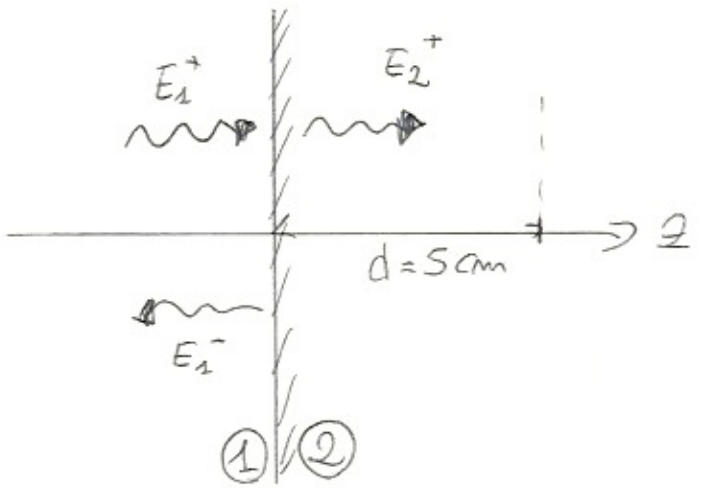
$$= 1{,}62 \cdot 10^{-14} \cdot e^{-j89{,}04^\circ} =$$

$$= 2{,}71 \cdot 10^{-16} - j\,1{,}62 \cdot 10^{-14} \quad [V/m]$$

~~EX 3~~

A PLANE WAVE AT FREQUENCY $f = 1$ GHz IMPINGES ORTHOGONALLY ON A DISCONTINUITY PLANE BETWEEN TWO DIELECTRICS WITH DIELECTRIC CONSTANT $\varepsilon_r = 1$ AND $\varepsilon_{r_2} = 4 - 0{,}4\,j$. THE AMPLITUDE OF THE INCIDENT FIELD IS $|E_1^+| = 2$ V/m.

CALCULATE THE AMPLITUDE OF THE ELECTRIC FIELD IN THE SECOND MEDIUM AT A DISTANCE OF $d = 5$ cm FROM THE INTERFACE.



TO CALCULATE THE ELECTRIC FIELD TRANSMITTED TO MEDIUM 2 WE NEED TO CALCULATE THE TRANSMISSION COEFFICIENT:

$$T = 1 + \Gamma \longrightarrow E_2^+(z) = E_2^+(0) \cdot e^{-\gamma_2 \cdot z}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad \gamma_2 = \alpha_2 - j\beta_2$$

IN FACT:

$$\frac{E_2^+}{E_1^+} = T \longrightarrow E_2^+ = T \cdot E_1^+ = (1 + \Gamma) \cdot E_1^+$$

LET'S START CALCULATING THE REFLECTION COEFFICIENT:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

① IS VACUUM SO $\eta_1 = \eta_0 = 377\,\Omega$

② $\eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_{r2}}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \sqrt{\frac{1}{\varepsilon_{r2}}} = 377 \cdot \sqrt{\frac{1}{4 - 0{,}4j}} = 377 \cdot \sqrt{\frac{1}{4{,}02 \cdot e^{-j5{,}71°}}}$

$$= 377 \cdot 0{,}5 \cdot e^{+j2{,}855°} = 188{,}5\, e^{+j2{,}885°} = 187{,}8 + j9{,}35\,\Omega$$

$$\Gamma = \frac{187{,}8 + j9{,}35 - 377}{187{,}8 + j9{,}35 + 377} = \frac{-189{,}2 + j9{,}35}{564{,}8 + j9{,}35} =$$

$$= \frac{189{,}43 \cdot e^{+j177{,}17°}}{564{,}8 \cdot e^{+j0{,}95°}} = 0{,}335 \cdot e^{+j176{,}22°} = -0{,}334 + j0{,}022$$

SO WE CAN GET THE TRANSMITTED FIELD AT THE INTERFACE:

$$E_2^+ = E_1^+ (1 + \Gamma) = 2 \cdot (1 - 0{,}334 + j0{,}022) =$$

$$= 2 \cdot (0{,}666 + j0{,}022) = 1{,}332 + j0{,}044 = 1{,}332 \cdot e^{+j1{,}89°} \; [V/m]$$

NOW WE NEED TO EVALUATE THE PROPAGATION CONSTANT OF THE SECOND MEDIUM:

$$\gamma_2 = j\omega \sqrt{\mu_0 \varepsilon_0 \varepsilon_{r2}} = e^{j90°} \cdot 2\pi \cdot 10^9 \cdot \left(\frac{1}{3 \cdot 10^8}\right) \sqrt{4{,}02 \cdot e^{-j5{,}71°}} =$$

$$= 2{,}1 + j41{,}95 = \alpha_2 + j\beta_2$$

SO WE HAVE:

$$E_2^+(z) = E_2^+(0) \cdot e^{-\gamma_2 \cdot z}$$

$$E_2^+(z=d) = E_2^+(0) \cdot e^{-\alpha_2 \cdot d} \cdot e^{-j\beta_2 d}$$

SUBSTITUTING THE FOLLOWING PARAMETERS IN THE ABOVE EXPRESSION WE GET:

$$\left[\begin{array}{l} \alpha_2 = 2,1 \\ \beta_2 = 41,95 \\ d = 5\ cm = 0,05\ m \end{array}\right]$$

$$E_2^+(z) = 1,332 \cdot e^{+j1,89^\circ} \cdot e^{-2,1 \cdot 0,05} \cdot e^{-j\overbrace{41,95 \cdot 0,05}^{rad}}$$

$$= 1,332 \cdot e^{-0,105} \cdot e^{j1,89^\circ} \cdot e^{-j120,18^\circ} =$$

$$= 1,199 \cdot e^{-j118,29^\circ} =$$

$$= 1,199 \cdot e^{-j2,065} \quad [V/m]$$