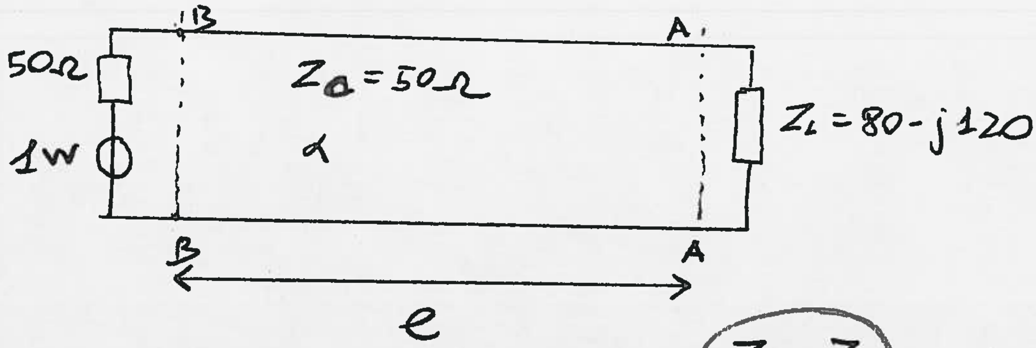
ESERCIZIO (Lossy transmission lines: power dissipation)

Calculated the power dissipated by the load in the case of

a) lossless transmission line ($\alpha = 0$)

b) lossy transmission line ($\alpha = 30$ dB/Km)



$Z_g = 50\Omega$

$Z_0 = 50\Omega$

$f = 300$ MHz

$\ell = 100$ m

$P_d = 1$ W

$Z_g = Z_0$

a) Lossless line ($\alpha = 0$)

Since $Z_0 = Z_g$, the full power available at the generator section (P_d) is transmitted to the line

$$\Gamma_g = 0 \Rightarrow P^+_{BB} = P_d\left(1 - |\Gamma_g|^2\right) = P_d$$

No power is dissipated by the line, so

$$P^+_{AA} = P^+_{BB} = P_d$$

At the load section ($Z_L \neq Z_0$) the reflection coefficient is

$$\Gamma_{AA} = \frac{Z_L - Z_0}{Z_L + Z_0} = 0{,}584 - j0.383 \qquad |\Gamma_A| = 0.7$$

Therefore the power dissipated by the load is

$$P_L = P_{Ass} = P_d\left(1 - |\Gamma_{AA}|^2\right) = 0.51 \text{ W}$$

b) Lossy case ($\alpha = 30$ dB/Km)

$$\alpha \left[{}^{Np}/_{m} \right] = \alpha \left[{}^{dB}/_{Km} \right] \frac{1}{8686} = 3{,}454 \cdot 10^{-3}\ Np/m$$

The reflection coefficient at the ~~load~~ generator section

$$|\Gamma_{BB}| = |\Gamma_{AA}|\, e^{-2\alpha \ell}$$

$$= 0.7\, e^{-2(3.454 \cdot 10^{-3}) \cdot 100} = 0.35$$

The total dissipated power (line + load) is

$$P_{ASS} = P_d \left(1 - |\Gamma_{BB}|^2\right) = 0.8775\ W$$

To calculate the power dissipated by the load, we need to evaluate the power available at section AA

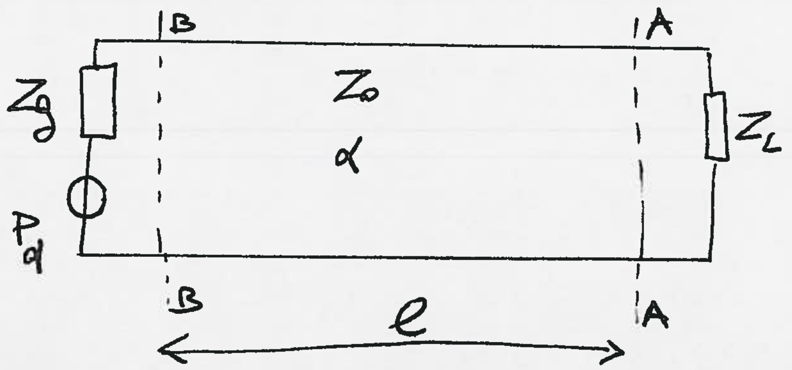
$$P^{+}_{AA} = P^{+}_{BB}\, e^{-2\alpha \ell} = P_d\, e^{-2\alpha \ell} = 0{,}5\ W$$

The power dissipated by the load is

$$P_L = P^{+}_{AA} \left(1 - |\Gamma_{AA}|^2\right) = 0{,}255\ W$$

EXERCISE | Power dissipation in lossy transmission line…

Calculate the power provided by the generator and the power dissipated by the load



$Z_g = 80\,\Omega$

$Z_0 = 50\,\Omega$

$\alpha = 60$ dB/100m

$Z_L = 50\,\Omega$

$\ell = 5$ m

$Z_0 = Z_L$

$$\alpha\left[{}^{N}/_{m}\right] = \alpha\left[{}^{dB}/_{m}\right]\frac{1}{8{,}686} = 0{,}0691 \text{ Np/m}$$

The reflection coefficient at the load section is

$$|\Gamma_g| = \left|\frac{Z_0 - Z_g}{Z_0 + Z_g}\right| = \left|\frac{50-80}{50+80}\right| = 0{,}231$$

The power P_{BB} provided by the generator to the line is

$$P_{BB} = P_d\left(1 - |\Gamma_g|^2\right) = 0{,}947 \text{ W}$$

The power dissipated by the load (P_L) is equal to the total power at section A (P_{AA}) because $\Gamma_{AA} = 0$ (the load is matched to the line impedance)

$$P_{AA} = P_{BB}\,e^{-2\alpha\ell} = 0{,}947\,e^{-2\cdot 0{,}0691\cdot 5}$$

= ~~0,437~~ W
0,475

ESERCIZIO (Power dissipation in lossy lines)

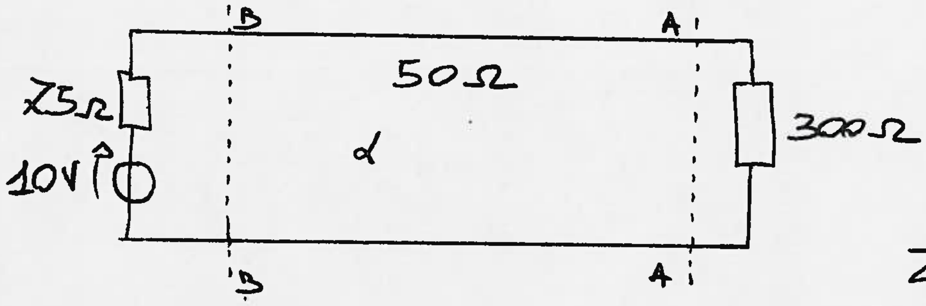
For the transmission line shown in the figure calculate

a) Power dissipated by the load for $\alpha = 0$

b) Power dissipated by the load for $\alpha = 20$ dB/Km

c) Power dissipated at the generator section ($\alpha = 20$ dB/Km)

d) Power dissipated by the line ($\alpha = 20$ dB/Km)



$f = 100$ MHz

$\lambda = 3$ m

$Z_g = 75\ \Omega$

$Z_0 = 50\ \Omega$

$Z_L = 300\ \Omega$

$Z_g \neq Z_0 \neq Z_L$

The available power provided by the generator is

$$P_d = \frac{|V_g|^2}{8\, Re[Z_g]} = \frac{100}{8 \cdot 75} = 0{,}167 \text{ W}$$

a) Lossless case ($\alpha = 0$)

We need to calculate the reflection coefficient Γ_g at the generator section

$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g}$$

The normalized impedance of the load is

$$\bar{z}_L = \frac{Z_L}{Z_0} = \frac{300}{50} = 6$$

Onto the Smith Chart we need to rotate $\bar{z}_L$ point by

$$\bar{\ell} = \frac{\ell}{\lambda} = \frac{100\,m}{3\,m} = \left(33 + \frac{1}{3}\right)\lambda$$

$\rightarrow \frac{1}{3}\lambda$ effective rotation

After this rotation, we stop at point

$$\bar{z}_{BB} = 0{,}22 + j\,0.55$$

that corresponds to $Z_{BB} = \bar{z}_{BB} Z_0 = 11 + j\,27{,}5\ [\Omega]$

The reflection coefficient at the generator is

$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g} = -0{,}5824 + j\,0{,}506$$

$$|\Gamma_g| = 0{,}77$$

$$P_L = P_a\left(1 - |\Gamma_g|^2\right) = 0{,}0675\ W$$

power absorbed by the load for $\alpha = 0$

b) Lossy case ($\alpha \neq 0$)

The Smith Chart can be still used, but the attenuation of the line must be taken into account.

$$\Gamma'_{BB} = \Gamma_{AA} e^{-2\gamma l} = \Gamma_{AA} \underbrace{\left(e^{-2\alpha l}\right)}_{\text{amplitude}} \underbrace{\left(e^{-j2\beta l}\right)}_{\text{phase}}$$

2 movements onto the Smith Chart

① ⟹ phase $e^{-j2\beta l}$ is a rotation with $|\Gamma| = \text{constant}$

② ⟹ amplitude $e^{-2\alpha l}$ is a movement toward the center of the smith chart

① The same rotation as in the case ($\alpha = 0$)

⟹ $\bar{z}_{BB}$

② $|\Gamma'_{BB}| = |\Gamma_{AA}| e^{-2\alpha l} = 0{,}7143 \cdot e^{-2(2{,}3 \cdot 10^{-3})100} = 0{,}45$

$$|\Gamma_{AA}| = \frac{Z_L - Z_0}{Z_L + Z_0} = 0{,}7143$$

$$\alpha\,[Np/m] = \frac{\alpha\,[dB/Km]}{8686} = 2{,}3 \cdot 10^{-3}\,[Np/m]$$

The point Γ'_{BB} correspond to a normalized impedance

$z'_{BB} = 0.48 + j\,0.48$

that is to

$$Z'_{BB} = \bar{z}'_{BB} Z_0 = 24 + j\,24\ [\Omega]$$

The reflection coefficient at the generator is thus

$$|\Gamma_g| = \left| \frac{Z'_{BB} - Z_g}{Z'_{BB} + Z_g} \right| = 0,55$$

The total power absorbed at the generator section is

$$P_{ASS} = P_d \left(1 - |\Gamma_g|^2\right) = 0,1156\ W$$

The forward propagating power at section B is

$$P_0^+ = \frac{P_{ASS}}{1 - |\Gamma'_{BB}|^2}$$

$$= \frac{0,1156\,W}{1 - |0.45|^2} = 0,145\ W$$

$$P_{ASS} = P_0^+ - P_0^-$$

$$|\Gamma_{BB}|^2 = \frac{P_0^-}{P_0^+}$$

$$P_{ASS} = P_0^+ - P_0^+ |\Gamma_{BB}|^2 = P_0^+ \left(1 - |\Gamma_{BB}|^2\right)$$

The forward propagating power at section A is

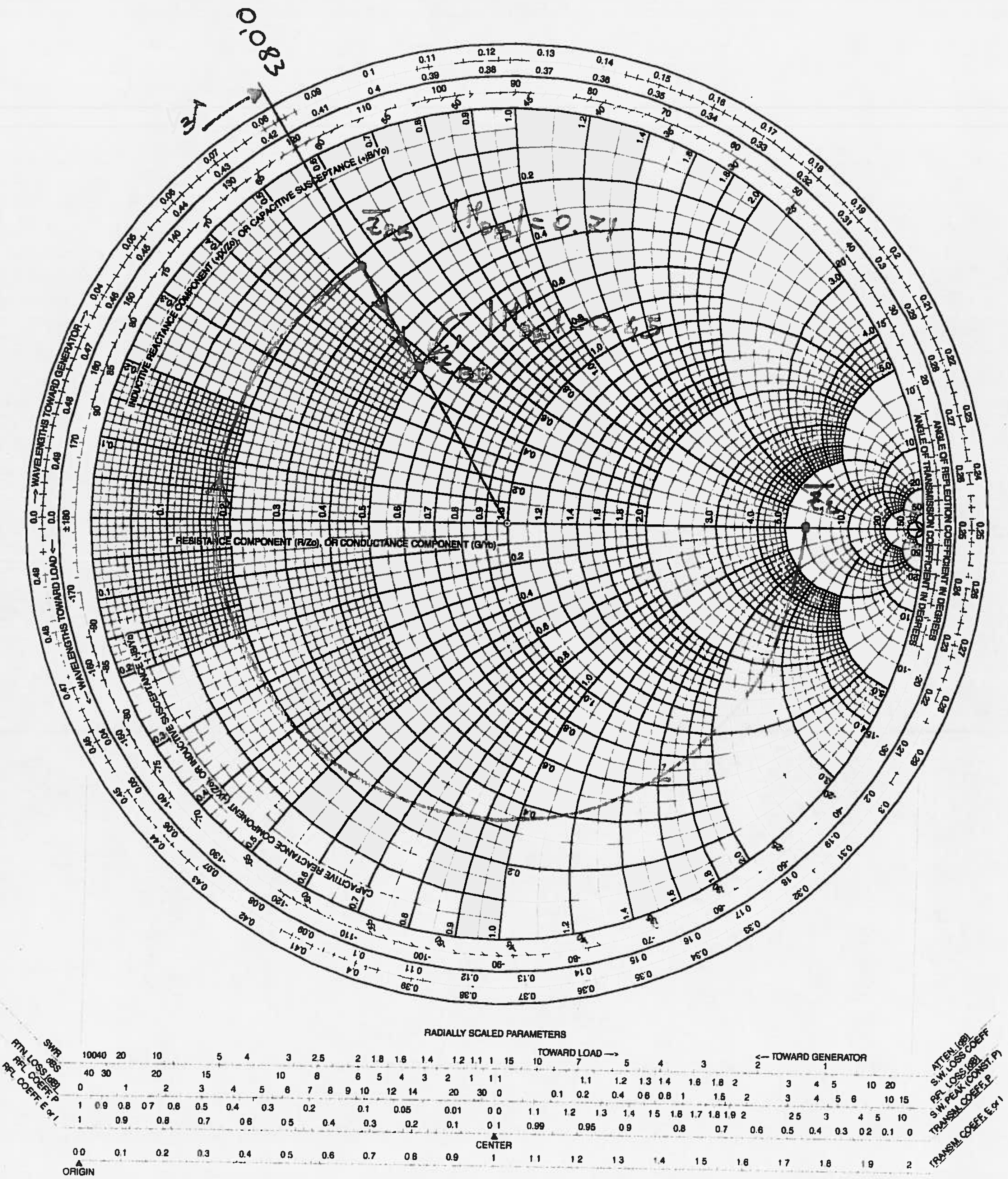
$$P_A^+ = P_0^+ e^{-2\alpha l} = 0.0915$$

The power absorbed by the load is

$$P_L = P_A^+ \left(1 - |\Gamma_{AA}|^2\right) = 0.045\ W$$

The power dissipated by the line is $P_{diss} = P_{ASS} - P_L = 0,07\ W$

Smith Chart



WAVELENGTHS TOWARD GENERATOR →
← WAVELENGTHS TOWARD LOAD
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo) OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS
TOWARD LOAD →
← TOWARD GENERATOR
SWR
dBS
RTN. LOSS [dB]
RFL. COEFF, P
RFL. COEFF, E or I
ATTEN. [dB]
S.W. LOSS COEFF
RFL. LOSS [dB]
S.W. PEAK (CONST. P)
TRANSM. COEFF, P
TRANSM. COEFF, E or I
CENTER
ORIGIN
0.083