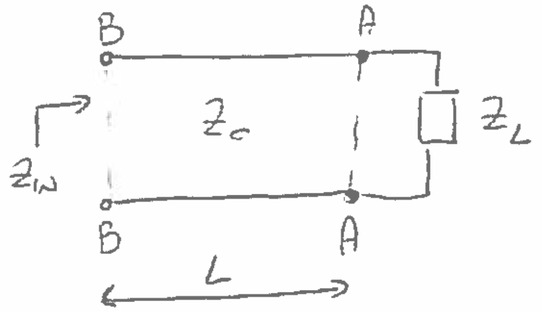
EX 1

ⓐ CALCULATE THE INPUT IMPEDENCE Z_{IN} AND THE REFLECTION COEFFICIENT Γ_{IN} AT THE INPUT OF THE TRANSMISSION LINE SHOWN IN THE PICTURE.

ⓑ AT WHAT DISTANCE FROM THE LOAD (Z_L) THE VOLTAGE IN THE LINE IS MAXIMUM / MINIMUM ?



$$Z_L = 50 + j\,150\ \Omega$$

$$Z_c = 75\ \Omega$$

$$L = 2{,}9\ m$$

$$f = 200\ MHz$$

$$\varepsilon_r = 1$$

ⓐ

FIRST OF ALL LET'S CALCULATE THE REFLECTION COEFFICIENT AT SECTION AA:

$$\Gamma(z=L) = \Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{50 + j150 - 75}{50 + j150 + 75} = \frac{-25 + j150}{125 + j150} =$$

$$= 0{,}78\ e^{j49{,}27^\circ}$$

WE CAN NOW EVALUATE Γ AT DIFFERENT SECTION:

$$\Gamma(z) = \Gamma_L \cdot e^{-j2\beta(L-z)}$$

where $\beta = \frac{2\pi}{\lambda}$ and $\lambda = \frac{c}{f\cdot\sqrt{\varepsilon_r}} = \frac{3\cdot 10^8}{200\cdot 10^6} = \frac{3}{2} = 1{,}5\ m$

AT SECTION BB

$$\Gamma_{IN} = \Gamma(z=0) = \Gamma_L \cdot e^{-j2\cdot\frac{2\pi}{\lambda}\cdot L} = \Gamma_L \cdot e^{-j4\pi\left(\frac{L}{\lambda}\right)}$$

NORMALIZED LENGTH OF THE LINE

$$\bar{L} = \frac{L}{\lambda} = \frac{2{,}9\ m}{1{,}5\ m} = 1{,}933 = 1{,}5 + 0{,}433$$

$$\Gamma_{IN} = \Gamma_L \cdot e^{-j4\pi(1,5+0,433)} = \Gamma_L \cdot e^{-j(\cancel{6\pi}+1,732\pi)} =$$

$$= \Gamma_L \cdot e^{-j5,44} = \Gamma_L \cdot e^{-j311,76^\circ} = 0,78 \cdot e^{j49,27^\circ} e^{-j311,76^\circ} =$$

$$= \boxed{0,78 \cdot e^{-262,49^\circ}} = 0,78 \cos(-262,49^\circ) + 0,78\, j \sin(-262,49^\circ) =$$

$$= -0,11 + j0,77$$

SO THE IMPEDENCE AT SECTION BB FOLLOWS:

$$Z_{IN} = Z_{in}(z=0) = Z_c \cdot \left(\frac{1+\Gamma_{IN}}{1-\Gamma_{IN}}\right) = 75 \cdot \left(\frac{1-0,11+j0,77}{1+0,11-j0,77}\right) =$$

$$= 75 \cdot \left(\frac{0,89+j0,77}{1,11-j0,77}\right) = 75 \cdot \left(\frac{1,18 \cdot e^{j40,86^\circ}}{1,35\, e^{-j34,7}}\right) = \boxed{16,38 + j63,5}\ [\Omega]$$

b) THE VOLTAGE AT SECTION z IS GIVEN BY THE SUMMATION OF INCIDENT AND REFLECTED WAVES:

$$V(z) = V^+(z) + V^-(z) = V^+(z)\left[1+\Gamma(z)\right] =$$

$$= V_0^+ \cdot e^{-j\beta z} \left[1 + \Gamma_L \cdot e^{-j2\beta(L-z)}\right] \qquad L - z = d$$

d IS THE DISTANCE FROM THE LOAD

$$|V(z)| = |V_0^+| \cdot \left|1 + \underbrace{|\Gamma_L| \cdot e^{j\varphi_L} \cdot e^{-j2\beta d}}_{\Gamma(z)}\right|$$

- IS MAXIMUM when $\Gamma(z)$ IS REAL AND POSITIVE:

IT HAPPENDS when $e^{j\varphi_L} \cdot e^{-j2\beta d} = +1 = e^{-j2k\pi}$

$$\boxed{\varphi_L - 2\beta d = -2k\pi}$$

THE CONDITION ON d IS THE FOLLOWING ONE:

$$2\beta d = \varphi_L + 2k\pi$$

$$2\,\frac{2\cancel{\pi}}{\lambda}d = \frac{49{,}27^\circ}{180^\circ}\cancel{\pi} + 2k\cancel{\pi}$$

$$d_{MAX} = \frac{\lambda}{4}\left[\frac{49{,}27}{180} + 2k\right] = \frac{1{,}5\ m}{4}\cdot\frac{49{,}27}{180} + k\cdot\frac{\lambda}{2} =$$

$$= \boxed{10{,}2\ cm + k\cdot\frac{\lambda}{2}}$$

with $k = 0, 1, 2, 3, \ldots$

- ON CONTRARY, $|V(z)|$ IS MINIMUM when:

$$\varphi_L - 2\beta d = -2(k+1)\pi$$

THIS CORRESPOND TO ADD $\frac{\lambda}{4}$ TO THE PREVIOUS d_{MAX}:

$$d_{MIN} = d_{MAX} + \frac{\lambda}{4} = 10{,}2\ cm + 37{,}5\ cm =$$

$$= \boxed{47{,}7\ cm + k\cdot\frac{\lambda}{2}}$$

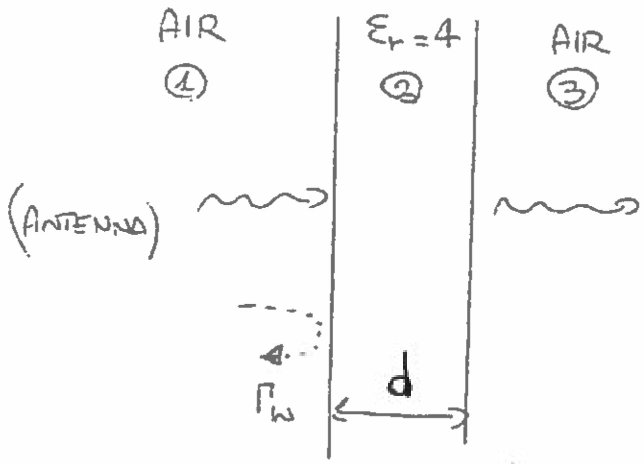
WITH $k = 0, 1, 2, 3, \ldots$

EX 2

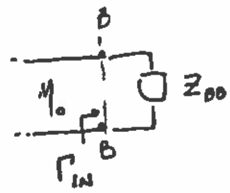
A RADAR ANTENNA IS COATED WITH A DIELECTRIC LAYER $(\varepsilon_r = 4)$ IN ORDER TO HAVE NO REFLECTIONS AT A FREQUENCY OF $f = 3\,GHz$.

a) CALCULATE THE THICKNESS OF THE LAYER

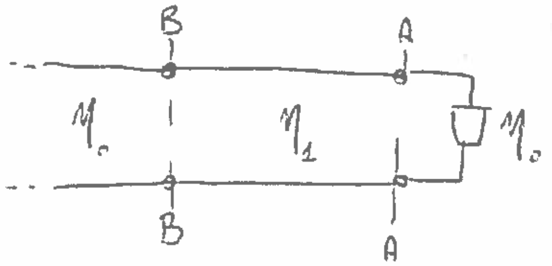
b) WHAT'S THE TRANSMITTED POWER IF THE ANTENNA IS USED AT $f' = 4\,GHz$?



HAVE NO REFLECTIONS MEANS THAT $\Gamma_{IN} = 0$



a) LET'S CONSIDER THE EQUIVALENT TRANSMISSION LINE



$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$$

$$\eta_1 = \frac{377\,\Omega}{\sqrt{\varepsilon_r}} = \frac{377}{2} = 188{,}5\ \Omega$$

IN ORDER TO HAVE NO REFLECTIONS, THE IMPEDENCE AT SECTION BB MUST BE EQUAL TO η_0, SO THE SAME VALUE OF THE IMPEDENCE AT AA SECTION. THIS IMPLIES THAT THE PROPAGATION THROUGH THE LINE MUST NO CHANGE THE IMPEDENCE VALUE

↓

$$\Gamma(z) = \Gamma_{AA} \cdot e^{-j2\beta(L-z)} = \Gamma_{AA} \cdot e^{-j2\beta d}$$

$$\Gamma_{BB} = \Gamma_{AA} \cdot e^{-j2\beta d} = 0{,}334 \cdot e^{-j2 \cdot \frac{2\pi}{\lambda} \cdot d}$$

$$\begin{cases} \beta = \frac{2\pi}{\lambda} \\ \lambda = \frac{c}{f \cdot \sqrt{\varepsilon_r}} = \frac{\cancel{3} \cdot \cancel{10^8}}{\cancel{3} \cdot 10^9 \cdot \sqrt{4}} = 5 \text{ cm} \\ \Gamma_{AA} = \frac{\eta_0 - \eta_1}{\eta_0 + \eta_1} = \frac{377 - 188{,}5}{377 + 188{,}5} = 0{,}331 \end{cases}$$

IT MEANS THAT THE REFLECTION AT SECTION AA DOESN'T CHANGE BECAUSE OF THE PROPAGATION ALONG THE LINE.

I MUST IMPOSE THAT $\Gamma_{BB} = \Gamma_{AA}$ THEREFORE:

$$\boxed{2 \cdot \frac{2\pi}{\lambda_2} \cdot d = 2k\pi}$$

$$\downarrow$$

$$d = k \cdot \frac{\lambda_2}{2} = \boxed{2{,}5 \text{ cm}}$$

(when $k=1$)

$\lambda_1 = \lambda_3 = 10$ cm

$\lambda_2 = \frac{10 \text{ cm}}{\sqrt{\varepsilon_r}} = 5$ cm

WAVELENGTH IN THE MEDIUM 2

SO I CAN VERIFY THAT $Z_{BB} = 2\eta_1 = \eta_0$

AND FROM IT FOLLOWS THAT $\Gamma_{IN} = \frac{2\eta_1 - \eta_0}{2\eta_1 + \eta_0} = \emptyset$

$\boxed{B}$ THE IMPEDENCE AT SECTION BB IS:

$$Z_{IN}(z) = Z_C \cdot \frac{1 + \Gamma_L \cdot e^{-2j\beta d}}{1 - \Gamma_L \cdot e^{-2j\beta d}}$$

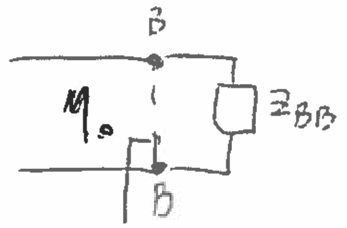
η_1

$$\cancel{2} \cdot \frac{2\pi \cdot \lambda_2}{\lambda_2' \cdot \cancel{2}} = 2\pi \cdot \frac{\lambda_2}{\lambda_2'} = 2\pi \cdot \frac{4}{3} = 8{,}37 = 120°$$

$$Z_{BB} = \eta_1 \cdot \frac{1 + 0{,}33 \cdot e^{-\boxed{2j\frac{2\pi}{\lambda_2'} \cdot \frac{\lambda_2}{2}}}}{1 - 0{,}33 \cdot e^{-2j\frac{2\pi}{\lambda_2'} \cdot \frac{\lambda_2}{2}}} =$$

$$= 188{,}5 \cdot \left(\frac{1 + 0{,}33 \; e^{-j120°}}{1 - 0.33 \cdot e^{-j120°}} \right) = 115 - j73{,}5 \; \Omega$$

where λ_2 AND λ_2' ARE THE WAVELENGTHS OF THE MEDIUM 2 AT $f = 3$GHz AND $f' = 4$GHz



$$\Gamma_{IN} = \frac{Z_{BB} - \eta_0}{Z_{BB} + \eta_0} = \frac{115 - j573,5 - 377}{115 - j573,5 + 377} =$$

$$= \boxed{0,55 \cdot e^{-j155^\circ}}$$

NOW Γ_{IN} IS NOT MORE EQUAL TO ZERO AS IT WAS IN THE CASE $\boxed{A}$

SO THE POWER TRANSMITTED THROUGH THE LAYER IS:

$$\frac{P_{TX}}{P_{IN}} = 1 - |\Gamma_{IN}|^2 = 1 - (0,55)^2 = 0,697$$

$$\downarrow$$

$$\boxed{69,7\,\%}$$