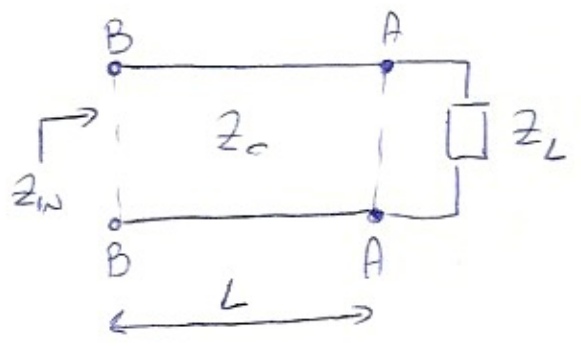


EX 1

① CALCULATE THE INPUT IMPEDENCE Z_{in} AND THE REFLECTION COEFFICIENT Γ_{in} AT THE INPUT OF THE TRANSMISSION LINE SHOWN IN THE PICTURE.

② AT WHAT DISTANCE FROM THE LOAD (Z_L) THE VOLTAGE IN THE LINE IS MAXIMUM/MINIMUM?



$$Z_L = 50 + j150 \Omega$$

$$Z_c = 75 \Omega$$

$$L = 2,9 \text{ m}$$

$$f = 200 \text{ MHz}$$

$$\epsilon_r = 1$$

②

FIRST OF ALL LET'S CALCULATE THE REFLECTION COEFFICIENT AT SECTION AA:

$$\Gamma(z=L) = \Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{50 + j150 - 75}{50 + j150 + 75} = \frac{-25 + j150}{125 + j150} = 0,78 \cdot e^{j59,27^\circ}$$

WE CAN NOW EVALUATE Γ AT DIFFERENT SECTION:

$$\Gamma(z) = \Gamma_L \cdot e^{-j2\beta(L-z)}$$

Where $\beta = \frac{2\pi}{\lambda}$ and $\lambda = \frac{c}{f \cdot \sqrt{\epsilon_r}} = \frac{3 \cdot 10^8}{200 \cdot 10^6} = \frac{3}{2} = 1,5 \text{ m}$

AT SECTION BB

$$\Gamma_{in} = \Gamma(z=0) = \Gamma_L \cdot e^{-j2 \cdot \frac{2\pi}{\lambda} \cdot L} = \Gamma_L \cdot e^{-j4\pi \cdot \left(\frac{L}{\lambda}\right)}$$

NORMALIZED LENGTH OF THE LINE

$$\bar{L} = \frac{L}{\lambda} = \frac{2,9 \text{ m}}{1,5 \text{ m}} = 1,933 = 1,5 + 0,433$$

$$\begin{aligned}
 \Gamma_w &= \Gamma_L \cdot e^{-54\pi(1,5 + 0,433)} = \Gamma_L \cdot e^{-5(\cancel{2} + 1,732\pi)} = \\
 &= \Gamma_L \cdot e^{-55,44} = \Gamma_L \cdot e^{-5311,76^\circ} = 0,78 \cdot e^{549,27^\circ} \cdot e^{-5311,76^\circ} = \\
 &= \boxed{0,78 \cdot e^{-262,49^\circ}} = 0,78 \cdot \cos(-262,49^\circ) + 0,78j \sin(-262,49^\circ) \\
 &= -0,11 + j0,77
 \end{aligned}$$

SO THE IMPEDENCE AT SECTION BB FOLLOWS:

$$\begin{aligned}
 Z_w &= Z_w(z=0) = Z_c \cdot \left(\frac{1 + \Gamma_w}{1 - \Gamma_w} \right) = 75 \cdot \left(\frac{1 - 0,11 + j0,77}{1 + 0,11 - j0,77} \right) = \\
 &= 75 \cdot \left(\frac{0,89 + j0,77}{1,11 - j0,77} \right) = 75 \cdot \left(\frac{1,18 \cdot e^{540,86^\circ}}{1,35 \cdot e^{-534,7^\circ}} \right) = \boxed{16,38 + j63,5} \\
 &\quad [\Omega]
 \end{aligned}$$

b) THE VOLTAGE AT SECTION z IS GIVEN BY THE SUMMATION OF INCIDENT AND REFLECTED WAVES:

$$\begin{aligned}
 V(z) &= V^+(z) + V^-(z) = V^+(z) [1 + \Gamma(z)] = \\
 &= V_c^+ \cdot e^{-j\beta z} \cdot [1 + \Gamma_L \cdot e^{-j2\beta(L-z)}]
 \end{aligned}$$

$L - z = d$ d IS THE DISTANCE FROM THE LEAD

$$|V(z)| = |V_c^+| \cdot \left| 1 + \underbrace{|\Gamma_L| \cdot e^{j\psi_L} \cdot e^{-j2\beta d}}_{\Gamma(z)} \right|$$

• IS MAXIMUM WHEN $\Gamma(z)$ IS REAL AND POSITIVE:

$$\Gamma \text{ HAPPENS WHEN } e^{j\psi_L} \cdot e^{-j2\beta d} = +1 = e^{-j2k\pi}$$

$$\boxed{\psi_L - 2\beta d = -2k\pi}$$

THE CONDITION ON d IS THE FOLLOWING ONE :

$$2\beta d = \psi_L + 2k\pi$$

$$2 \cdot \frac{2\pi}{\lambda} d = \frac{49,27^\circ}{180^\circ} \pi + 2k\pi$$

$$d_{\max} = \frac{\lambda}{4} \left[\frac{49,27}{180} + 2k \right] = \frac{1,5 \text{ m}}{4} \cdot \frac{49,27}{180} + k \cdot \frac{\lambda}{2} =$$
$$= \boxed{10,2 \text{ cm} + k \cdot \frac{\lambda}{2}}$$

with $k = 0, 1, 2, 3, \dots$

• ON CONTRARY, $|V(z)|$ IS MINIMUM WHEN :

$$\psi_L = 2\beta d = -2(k+1)\pi$$

THIS CORRESPOND TO ADD $\frac{\lambda}{4}$ TO THE PREVIOUS d_{\max} :

$$d_{\min} = d_{\max} + \frac{\lambda}{4} = \dots = 10,2 \text{ cm} + 37,5 \text{ cm} =$$

$$= \boxed{47,7 \text{ cm} + k \cdot \frac{\lambda}{2}}$$

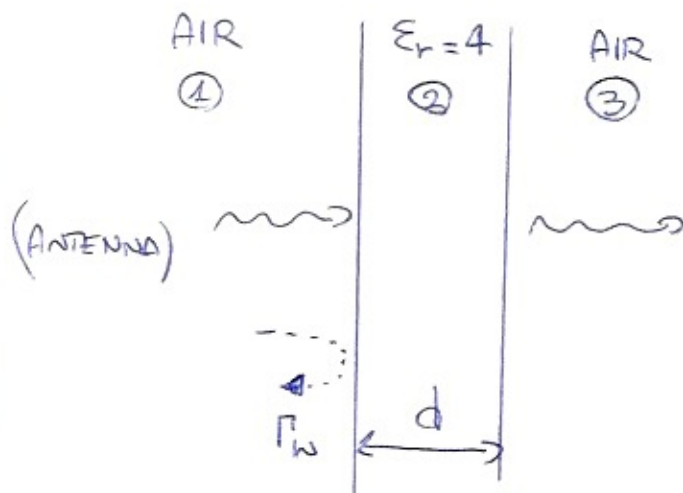
with $k = 0, 1, 2, 3, \dots$

EX 2

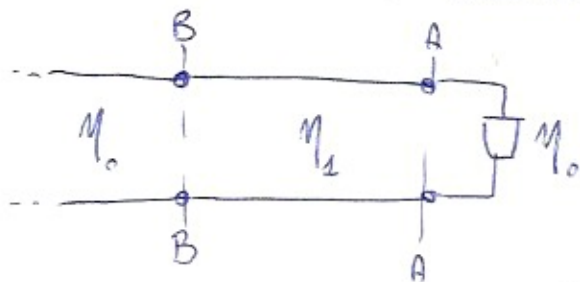
A RADAR ANTENNA IS COATED WITH A DIELECTRIC LAYER ($\epsilon_r = 4$) IN ORDER TO HAVE NO REFLECTIONS AT A FREQUENCY OF $f = 3 \text{ GHz}$.

a) CALCULATE THE THICKNESS OF THE LAYER

b) WHAT IS THE TRANSMITTED POWER IF THE ANTENNA IS USED AT $f' = 4 \text{ GHz}$?



c) LET'S CONSIDER THE EQUIVALENT TRANSMISSION LINE



$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \Omega$$

$$\eta_1 = \frac{377 \Omega}{\sqrt{\epsilon_r}} = \frac{377}{2} = 188,5 \Omega$$

IN ORDER TO HAVE NO REFLECTIONS, THE IMPEDENCE AT SECTION BB MUST BE EQUAL TO η_0 , SO THE SAME

VALUE OF THE IMPEDENCE AT AA SECTION. THIS IMPLIES THAT THE PROPAGATION THROUGH THE LINE MUST NOT CHANGE THE IMPEDENCE VALUE



$$\Gamma(z) = \Gamma_{AA} \cdot e^{-j2\beta(L-z)} = \Gamma_{AA} \cdot e^{-j2\beta d}$$

$$\Gamma_{BB} = \Gamma_{AA} \cdot e^{-j2\beta d} = 0,334 \cdot e^{-j2 \cdot \frac{2\pi}{\lambda} \cdot d}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{c}{f \cdot \sqrt{\epsilon_r}} = \frac{3 \cdot 10^8}{3 \cdot 10^9 \cdot \sqrt{4}} = 5 \text{ cm}$$

$$\Gamma_{AA} = \frac{\eta_0 - \eta_1}{\eta_0 + \eta_1} = \frac{377 - 188,5}{377 + 188,5} = 0,334$$

I MUST IMPOSE THAT $\Gamma_{BB} = 1$ THEREFORE:

$$\boxed{2 \cdot \frac{2\pi}{\lambda} \cdot d = 2k\pi}$$

$$\downarrow$$

$$d = k \cdot \frac{\lambda}{2} = \boxed{2,5 \text{ cm}}$$

$$\lambda = 5 \text{ cm}$$

$$\lambda' = 3,75 \text{ cm}$$

B THE IMPEDENCE AT SECTION **BB** IS:

$$Z_{in}(z) = \underbrace{Z_0}_{\eta_1} \cdot \frac{1 + \Gamma_L \cdot e^{-2j\beta d}}{1 - \Gamma_L \cdot e^{-2j\beta d}}$$

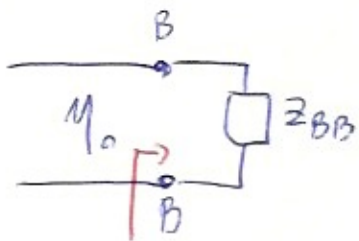
$$Z_{BB} = \eta_1 \cdot \frac{1 + 0,33 \cdot e^{-2j5 \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}}}{1 - 0,33 \cdot e^{-2j5 \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}}} =$$

$$= 188,5 \cdot \left(\frac{1 + 0,33 \cdot e^{-j120^\circ}}{1 - 0,33 \cdot e^{-j120^\circ}} \right) = 115 - j73,5 \ \Omega$$

$$\cancel{2} \cdot \frac{2\pi \cdot \lambda}{\lambda' \cdot \cancel{2}} = \frac{2\pi \cdot \lambda}{\lambda'}$$

$$= 2\pi \cdot \frac{4}{3} = 8,37 =$$

$$= 120^\circ$$



$$\Gamma_w = \frac{Z_{BB} - Z_0}{Z_{BB} + Z_0} = \frac{115 - 573,5 - 377}{115 - 573,5 + 377} =$$

$$= \boxed{0,55 \cdot e^{-5155^\circ}}$$

SO THE POWER TRANSMITTED THROUGH THE LAYER IS:

$$\frac{P_{TX}}{P_w} = 1 - |\Gamma_w|^2 = 1 - (0,55)^2 = 0,697$$



$$\boxed{69,7\%}$$