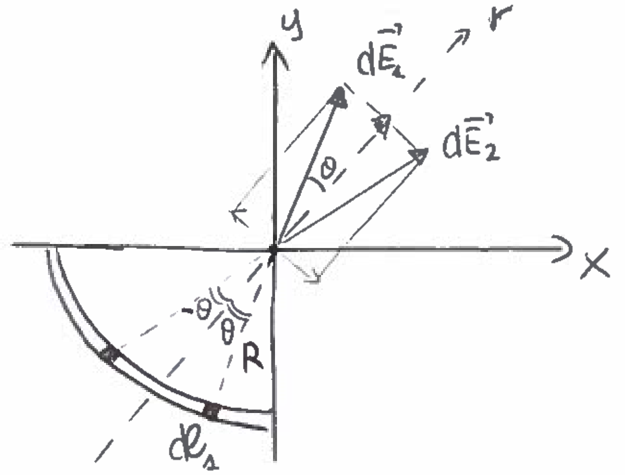
EX 1

A QUARTER CIRCLE OF RADIUS R HAS UNIFORM LINEAR POSITIVE CHARGE DENSITY ρ_ℓ. CALCULATE THE TOTAL ELECTRIC FIELD GENERATED BY THIS DISTRIBUTION IN THE ORIGIN.



$$\rho_\ell = \lim_{\Delta L \to 0} \frac{Q}{\Delta L}$$

DEFINITION OF LINEAR CHARGE DENSITY: THE TOTAL CHARGE IS SPREAD ON THIS LINE.

$$d\vec{E}_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dQ}{R^2} \vec{\mu}_r \qquad dQ = \rho_\ell \cdot d\ell$$

$$d\vec{E} = (dE_1 \cdot \cos\theta) \cdot 2 \cdot \vec{\mu}_r = \frac{\cancel{2}}{{}_2\cancel{4}\pi\varepsilon_0} \cdot \frac{\rho_\ell \cdot d\ell}{R^2} \cdot \cos\theta\, \vec{\mu}_r = \underbrace{\boxed{\frac{\rho_\ell \cdot d\ell \cdot \cos\theta}{2\pi\varepsilon_0 R^2}}}_{dE} \vec{\mu}_r$$

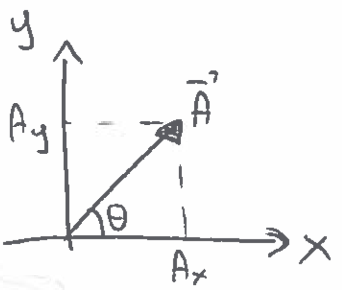
$$E = \int_\ell dE = \int_\ell \frac{\rho_\ell \cdot d\ell \cdot \cos\theta}{2\pi\varepsilon_0 \cdot R^2} = \int \frac{\rho_\ell \cdot (d\theta \cdot \cancel{R}) \cdot \cos\theta}{2\pi\varepsilon_0 \cdot R^{\cancel{2}}} = \frac{\rho_\ell}{2\pi\varepsilon_0 R}\int_0^{\pi/4} \cos\theta\, d\theta =$$

$$(d\ell = d\theta \cdot R)$$

$$= \frac{\rho_\ell}{2\pi\varepsilon_0 \cdot R}\left[\sin\theta\right]_0^{\pi/4} = \boxed{\frac{\sqrt{2} \cdot \rho_\ell}{4\pi\varepsilon_0 \cdot R}}$$

$$\vec{E} = E\, \vec{\mu}_r = \frac{\sqrt{2} \cdot \rho_\ell}{4\pi\varepsilon_0 \cdot R} \vec{\mu}_r$$

→ I WANT TO WRITE THIS VECTOR IN CARTESIAN COORDINATES



$$\vec{A} = A_x \vec{\mu}_x + A_y \vec{\mu}_y$$

$$A_x = |\vec{A}| \cdot \cos\theta$$

$$A_y = |\vec{A}| \cdot \sin\theta$$

IN THIS CASE $\vec{A} = \vec{\mu}_r$

$|\vec{A}| = 1 \quad \theta = 45°$

$$\vec{A} = \frac{\sqrt{2}}{2}\vec{\mu}_x + \frac{\sqrt{2}}{2}\vec{\mu}_y = \vec{\mu}_r$$

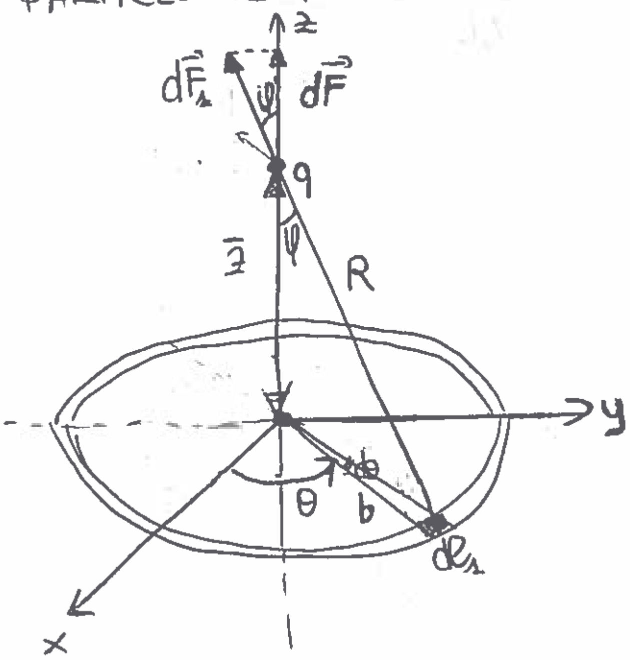
↓

$$\boxed{\vec{E} = \frac{\rho_\ell}{4\pi\varepsilon_0 R}\left(\vec{\mu}_x + \vec{\mu}_y\right)}$$

EX 2

A PARTICLE WITH A MASS m AND A CHARGE q $(q>0)$ IS PLACED ALONG THE AXIS OF A RING WITH A UNIFORM LINEAR CHARGE DENSITY ρ_ℓ (>
THE RADIUS OF THE RING IS b.

GIVEN THE DISTANCE z FROM THE RING CENTER AT WHICH THE PARTICLE IS AT EQUILIBRIUM, CALCULATE THE VALUE OF THE CHARGE q.



BEING AT EQUILIBRIUM MEANS THAT THE TOTAL FORCE ACTING ON PARTICLE IS ZERO.
HERE WE HAVE 2 FORCES:
↳ GRAVITY FORCE because particle has mass $\neq 0$
↳ ELECTRIC FORCE because particle is charged.

N.B. THE RING LAYS ON xy PLANE

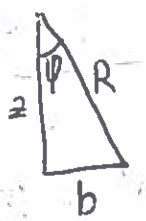
- GRAVITY FORCE $\vec{F}_g = -(mg)\vec{u}_z$

- ELECTROSTATIC FORCE: We first evaluate the contribution given by the infinitesimal portion $d\ell$ of the ring

$$d\vec{F}_1 = \underbrace{\boxed{\frac{1}{4\pi\varepsilon_0}\cdot\frac{dQ\cdot q}{R^2}}}_{dF_1}\vec{u}_r$$

WE NOTICE THAT:

1. $dQ = \rho_\ell \cdot d\ell_1 = \rho_\ell (b\cdot d\theta)$

2. 

$R = \sqrt{z^2+b^2}$
$\cos\varphi = \frac{z}{R}$

$$dF = (dF_1 + dF_2)\cdot\cos\varphi =$$
$$= 2dF_1\cdot\cos\varphi =$$
$$= 2\cdot\left[\frac{1}{4\pi\varepsilon_0}\cdot\frac{dQ\cdot q}{R^2}\right]\cos\varphi$$
$$= 2\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{\rho_\ell\cdot b\cdot d\theta\cdot q}{(z^2+b^2)}\cdot\frac{z}{R} =$$
$$= \frac{\rho_\ell\cdot q}{2\pi\varepsilon_0}\cdot\frac{bz}{(z^2+b^2)^{3/2}}\,d\theta$$

THE TOTAL CHARGE OF THE RING IS:

$$\boxed{Q = \rho_\ell \cdot L_{circle} = \rho_\ell \cdot 2\pi b}$$

$$F_z = \int_{\theta=0}^{\pi} dF = \int_{\theta=0}^{\pi} \frac{\rho_\ell \cdot q}{2\pi\varepsilon_0} \cdot \frac{b \cdot z}{(z^2+b^2)^{3/2}} \, d\theta = \frac{\rho_\ell \cdot q}{2\pi\varepsilon_0} \cdot \frac{b \cdot z}{(z^2+b^2)^{3/2}} \cdot \int_0^{\pi} d\theta =$$

$$= \frac{\rho_\ell \cdot q}{2\pi\varepsilon_0} \cdot \frac{b\, z\, \pi}{(z^2+b^2)^{3/2}} \cdot \frac{2}{2} = \frac{Q \cdot q}{4\pi\varepsilon_0} \cdot \frac{z}{(z^2+b^2)^{3/2}}$$

$$\vec{F}_e = F \vec{u}_z = \frac{Q \cdot q}{4\pi\varepsilon_0} \cdot \frac{z}{(z^2+b^2)^{3/2}} \vec{u}_z$$

THE PARTICLE IS AT EQUILIBRIUM IF:

$$\vec{F}_g + \vec{F}_e = 0$$

$$-mg\,\vec{u}_z + \frac{Qq}{4\pi\varepsilon_0} \cdot \frac{z}{(z^2+b^2)^{3/2}} \vec{u}_z = 0$$

$$\frac{Q\, q}{4\pi\varepsilon_0} \cdot \frac{z}{(z^2+b^2)^{3/2}} = m \cdot g$$

$$q = \frac{4\pi \cdot \varepsilon_0 \cdot m \cdot g}{Q} \cdot \frac{(b^2+z^2)^{3/2}}{z}$$

$$\left(\text{N.B} \quad 1\,F = \frac{C}{V} \right)$$

GIVEN THE FOLLOWING VALUES:

$\varepsilon_0 = 8{,}854 \cdot 10^{-12}\ F/m$

$m = 10^{-9}\ g$

$g = 9{,}8\ \frac{N}{Kg}$

$Q = 0{,}1\ \mu C$

$b = 1\ m$

$z = 9\ m$

$\rightarrow$ YOU GET:

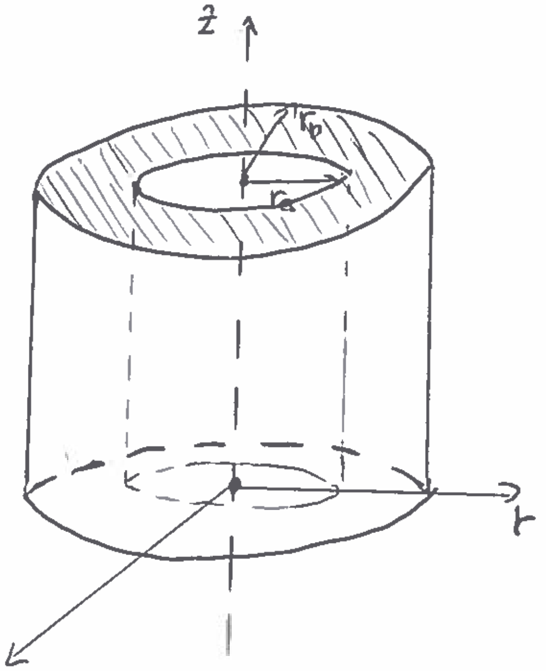
$$q = 12 \cdot 10^{-15}\ C$$

$$= 12\ fC$$

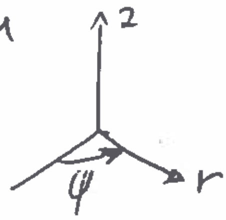
Ex 3

A CYLINDRICAL SHELL WITH INNER RADIUS r_a AND OUTER RADIUS r_b IS FILLED WITH A UNIFORM VOLUME CHARGE DENSITY ρ_v.

CALCULATE THE ELECTRIC FIELD $\vec{E}$ AND THE POTENTIAL EVERYWHERE IN THE SPACE.



- NOW WE ARE USING A CYLINDRICAL COORDINATE SYSTEM



- GIVEN THAT THE CYLINDRICAL SHALL HAS ∞ EXTENSION ALONG z DIRECTION, FOR SYMMETRY OF OUR GEOMETRY WE CAN SAY THAT THE FIELD ORIGINATED BY THE DISTRIBUTION DEPENDS JUST ON r, THE DISTANCE FROM THE CYLINDER AXIS.

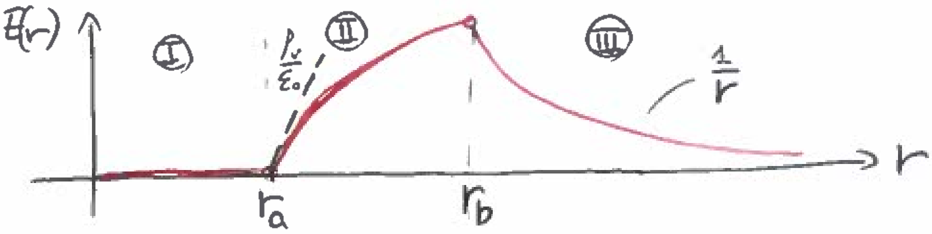
$$\vec{E} = E_r \vec{u}_r + \cancel{E_z \vec{u}_z} + \cancel{E_\varphi \vec{u}_\varphi}$$

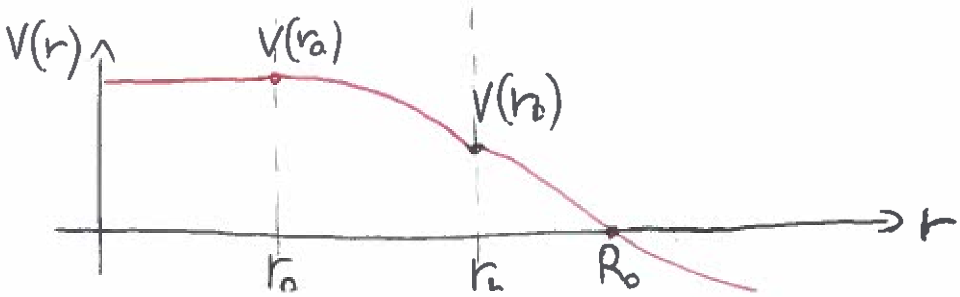
SO WE IDENTIFY 3 REGIONS OF THE SPACE:

(I) $r < r_a$ (IN THE 1ST CYLINDER)

(II) $r_a < r < r_b$ (IN THE CHARGED SHELL)

(III) $r > r_b$ (OUTSIDE THE SHELL)





WE START FROM THE FIRST REGION:

Ex 3 (PAG 2)

I $r < r_a$. WE APPY GAUSS LAW:

$$\oint_S \vec{D} \cdot d\vec{s} = Q_{INT}$$

IT STATES THAT THE FLUX OF THE ELECTRIC DENSITY $\vec{D}$ OVER A CLOSED SURFACE IS EQUAL TO ALL THE CHARGES INSIDE AT THAT SURFACE

Remember: $\vec{D} = \varepsilon \vec{E} = \varepsilon_0 \vec{E}$ ← IN VACUUM

SO WE HAVE TO IMAGINE TO SHAPE A SURFACE S (CYLINDRIC) WITH RADIUS $r < r_a$.

SINCE THERE ARE NO CHARGED ELEMENTS INSIDE S WE CAN SAY THAT $Q_{INT} = \emptyset$:

$$\oint_S \varepsilon_0 \vec{E} \cdot d\vec{s} = 0 \longrightarrow \boxed{E(r) = 0}$$

II $r_a < r < r_b$ AGAIN WE DESIGN A SURFACE S WITH RADIUS r (WITH $r_a < r < r_b$) AND WE APPLY GAUSS LAW:

$$\oint_S \vec{D} \cdot d\vec{s} = Q_{INT}$$

$$\vec{D} = \varepsilon_0 \vec{E} = \varepsilon_0 \cdot E_r \vec{u}_r$$

$$\underbrace{d\vec{s}}_{\text{SURFACE ELEMENT}} = ds \cdot \vec{u}_r$$

($\perp$ TO S)

$Q_{INT} = \rho_v \cdot \Delta V$ → PORTION OF VOLUME WITH CHARGE: IS THE VOLUME BETWEEN THE CYLINDER r_a and the CYL. WITH r.

$$\Delta V = V_2 - V_1 =$$
$$= \pi r^2 z - \pi r_a^2 z =$$
$$= \pi z (r^2 - r_a^2)$$

$$Q_{INT} = \rho_v \cdot \pi \cdot z (r^2 - r_a^2)$$

$$\oint_S \varepsilon_0 E_r \, ds = \rho_v \cdot \pi z (r^2 - r_a^2)$$

$$\varepsilon_0 E_r \cdot S_{ce.} = \rho_v \cdot \pi z (r^2 - r_a^2)$$

$$\varepsilon_0 E_r \cdot (2\cancel{\pi} r \cancel{z}) = \rho_v \cdot \cancel{\pi} \cdot \cancel{z} (r^2 - r_a^2)$$

$$\boxed{E(r) = \frac{\rho_v}{2\varepsilon_0} \frac{(r^2 - r_a^2)}{r}}$$

SO NOW I CAN FIND:

$$E(r_a) = 0$$

$$E(r_b) = \frac{\rho_v}{2\varepsilon_0} \left(\frac{r_b^2 - r_a^2}{r_b} \right)$$

THE SLOPE OF E_r IS GIVEN BY:

$$\frac{\partial E(r)}{\partial r} = \frac{\rho_v}{2\varepsilon_0} \left(\frac{r^2 + r_a^2}{r^2} \right)$$

↓

$$\left. \frac{\partial E(r)}{\partial r} \right|_{r = r_a} = \frac{\rho_v}{\varepsilon_0}$$

$\boxed{I}$ $r > r_b$ AGAIN GAUSS LAW (SAME PROCEDURE)

$$\int_S \vec{D} \cdot d\vec{s} = Q_{INT}$$

$$E_r \cdot \varepsilon_0 \int_S ds = V_{SHELL} \cdot \rho_v$$

$$E_r \cdot \varepsilon_0 (2\pi r z) = z\pi^2 (r_b^2 - r_a^2) \cdot \rho_v$$

$$E(r) = \frac{\pi^{\cancel{2}} (r_b^2 - r_a^2) \cdot \rho_v \cdot \cancel{z}}{2 \cancel{\pi} r \cancel{z} \cdot \varepsilon_0} = \boxed{\frac{\rho_v \cdot \pi}{2\varepsilon_0} \cdot \frac{(r_b^2 - r_a^2)}{r}}$$

So $E(r) \propto \frac{1}{r}$

POTENTIAL : NOW WE GO IN THE OPPOSITE DIRECTION, SO WE START FROM POTENTIAL IN LAST REGION III.

REMEBER THAT THE POTENTIAL IN A POINT P IS GIVEN BY THIS FORMULA:

$$V(P) = -\int_{P_0}^{P} \vec{E} \cdot d\vec{e} + V_0$$

→ IS ALWAYS RELATED TO THE POTENTIAL OF ANOTHER POINT, P_0, WHERE WE KNOW THAT $\underline{V(P_0) = V_0}$

→ IS THE INTEGRAL OF THE ELECTRIC FIELD OVER A LINE FROM P_0 TO P

III $r > r_b$

$$V(r) = -\int_{R_0}^{r} \vec{E} \cdot d\vec{e} + \overbrace{V(R_0)}^{0} = -\int_{R_0}^{r} E_r \, dr = -\int_{R_0}^{r} \frac{\rho_v \pi}{2\varepsilon_0} \frac{(r_b^2 - r_a^2)}{r} dr =$$

EXPRESS. OF E_r IN REGION III

WE SET $R_0 \gg r_b$, IN WHICH WE DEFINE $V_0 = \emptyset$

$$d\vec{e} = dr\,\vec{u}_r + \cancel{r\,d\theta\,\vec{u}_\theta} + \cancel{dz\,\vec{u}_z}$$

IN CYL. COORDINATES IS, BUT WE CARE ONLY OF MOVEMENT ALONG $\vec{u}_r$, BECAUSE $\vec{E} = E_r \vec{u}_r$

$$= -\frac{\rho_v \pi}{2\varepsilon_0} (r_b^2 - r_a^2) \int_{R_0}^{r} \frac{1}{r} dr =$$

$$= -\frac{\rho_v \pi}{2\varepsilon_0} (r_b^2 - r_a^2) \left[\ln(r)\right]_{R_0}^{r} =$$

$$= \boxed{\frac{\rho_v \pi}{2\varepsilon_0} (r_b^2 - r_a^2) \cdot \ln\left(\frac{R_0}{r}\right)}$$

so $V(r_b) = \frac{\rho_v \pi}{2\varepsilon_0}(r_b^2 - r_a^2) \cdot \ln\left(\frac{R}{r_b}\right)$

$\boxed{\text{II}}$ $r_a < r < r_b$

$$V(r) = -\int_{r_b}^{r} E(r)\Big|_{\text{II}} dr + V(r_b) = -\int_{r_b}^{r} \frac{\rho_v}{2\varepsilon_0} \frac{(r^2 - r_a^2)}{r} dr + V(r_b) =$$

HERE I TAKE AS STARTING POINT r_b, BECAUSE I KNOW $V(r_b)$

$$= -\frac{\rho_v}{2\varepsilon_0}\int_{r_b}^{r}\left(r - \frac{r_a^2}{r}\right) dr + V(r_b) =$$

$$= -\frac{\rho_v}{2\varepsilon_0}\left[\frac{r^2}{2} - r_a^2 \ln(r)\right]_{r_b}^{r} + V(r_b) =$$

$$= \boxed{\frac{\rho_v}{2\varepsilon_0}\left[\frac{r_b^2 - r^2}{2} - r_a^2 \ln\left(\frac{r_b}{r}\right)\right] + V(r_b)}$$

$\boxed{\text{III}}$ $r < r_a$

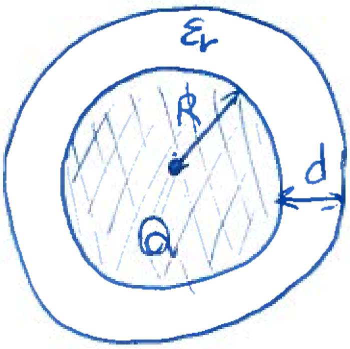
$$V(r) = -\int_{r_a}^{r} \underbrace{E(r)}_{\emptyset} dr + V(r_a) = V(r_a)$$

$$V(r) = \boxed{\frac{\rho_v}{2\varepsilon_0}\left[\frac{r_b^2 - r_a^2}{2} - r_a^2 \ln\left(\frac{r_b}{r_a}\right)\right] + V(r_b)} = \text{COST}$$

EX 4

A METAL SPHERE WITH A RADIUS R AND A CHARGE Q IS SURROUNDED BY A LAYER OF HOMOGENEOUS AND ISOTROPIC DIELECTRIC, WITH DIELECTRIC CONSTANT ε_r AND THICKNESS d.

- CALCULATE THE ELECTRIC FIELD $\vec{E}$ AND POTENTIAL V EVERYWHERE IN SPACE
- CALCULATE THE BOUND SURFACE CHARGE DENSITY ρ_{S_b} AND THE BOUND VOLUME CHARGE DENSITY ρ_{V_b} INDUCED BY THE DIELECTRIC. → NO



3 REGIONS:

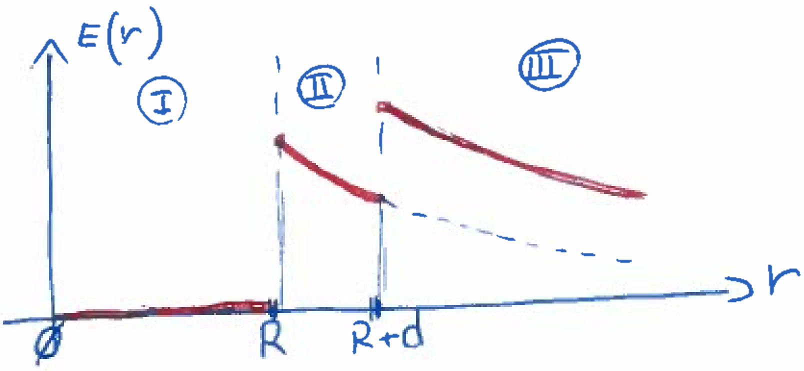
$\boxed{I}$ $r < R$ (CONDUCTOR)

$\boxed{II}$ $R < r < R+d$ (DIELECTRIC)

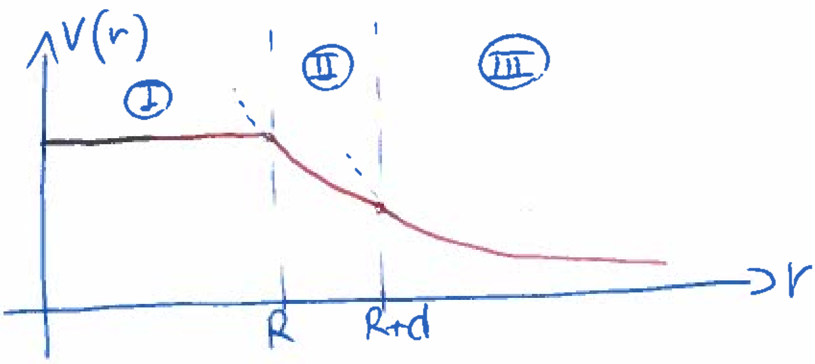
$\boxed{III}$ $r > R+d$ (VACUUM)

$\boxed{I}$ $r < R$

BY DEFINITION THE ELECTRIC FIELD IN A CONDUCTOR IS ALWAYS $\varnothing$. $\boxed{E(r)=0}$



ANYWAY, THE GEOMETRY IS A SPHERE, SO EVERYTHING IS SYMMETRIC, SO THE EL. FIELD IS DEPENDENT ONLY ON r, THE DISTANCE FROM DISTRIBUTION (NOT θ AND φ dependence):



$$\vec{E}(r,\theta,\varphi) = E(r)\,\vec{u}_r$$

II] $R < r < R+d$ $\rightarrow Q$

GAUSS LAW: $\oint_S \vec{D} \cdot d\vec{s} = Q_{INT}$

S is a sphere with radius r

$$\oint_S (\varepsilon \cdot E_r \vec{u}_r) \cdot (ds\, \vec{u}_r) = \varepsilon_0 \cdot \varepsilon_r \cdot E_r \overbrace{\oint_S dS}^{\text{SUP. SFERA RAGGIO R}} = \varepsilon_0 \cdot \varepsilon_r \cdot E_r \cdot 4\pi r^2$$

$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$

$$\varepsilon_0 \varepsilon_r \cdot E(r) \cdot 4\pi r^2 = Q$$

$$\boxed{E(r) = \frac{Q}{\varepsilon_0 \varepsilon_r 4\pi r^2}}$$

$\rightarrow$ AT THE DISCONTINUITY BETWEEN DIEL. AND VACUUM: $r = (R+d)$

$$E(R+d) = \frac{Q}{\varepsilon_0 \varepsilon_r 4\pi (R+d)^2}$$

III] $\oint_S \vec{D} \cdot d\vec{s} = Q_{INT} = Q$

$$\varepsilon_0 E_r 4\pi r^2 = Q$$

$$\boxed{E(r) = \frac{Q}{\varepsilon_0 4\pi r^2}}$$

$\rightarrow$ $E(R+d) = \dfrac{Q}{\varepsilon_0 4\pi (R+d)^2}$

SO I SEE THAT

$E(R+d-\delta) \neq E(R+d+\delta)$

ELECTRIC FIELD IS NOT CONSERVATIVE: THE VECTOR D IS CONSERVING

$D^- = \varepsilon \cdot E^- = \varepsilon_0 \cdot \varepsilon_r \cdot E^-$

$D^+ = \varepsilon \cdot E^+ = \varepsilon_0 \cdot E^+$

$\boxed{D^- = D^+}$

POTENTIAL

III] $$V(r) = \int_{r=\infty}^{r} -E_r\, dr + \overbrace{V(\infty)}^{0} =$$

(HERE I CAN SAY THAT $r \to \infty$ WHERE $V(\infty) = 0$)

$$= \int_{\infty}^{r} -\frac{Q}{\varepsilon_0 4\pi r^2} dr =$$

$$= \boxed{\frac{Q}{4\pi \varepsilon_0 r}}$$

I] $R < r < R+d$

$$V(r) = -\int_{R+d}^{r} E_{II}\, dr + V(R+d) = -\int_{R+d}^{r} \frac{Q}{\varepsilon_0 \varepsilon_r 4\pi r^2} dr + \frac{Q}{4\pi \varepsilon_0 (R+d)} =$$

$$= \frac{Q}{4\pi \varepsilon_0 \varepsilon_r}\left[\frac{1}{r} - \frac{1}{R+d}\right] + \frac{Q}{4\pi \varepsilon_0 (R+d)}$$

$\boxed{III}$ $r < R$

$$V(r) = -\int_R^r \underset{0}{\underset{\shortparallel}{E_r}}\, dr + V(R) = V(R) = \frac{Q}{4\pi\varepsilon_0\varepsilon_r}\left[\frac{1}{R} - \frac{1}{R+d}\right] + \frac{Q}{4\pi\varepsilon_0(R+d)}$$

$$= \boxed{\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\varepsilon_r R} + \frac{\varepsilon_r - 1}{(R+d)\varepsilon_r}\right]}$$

$$= \text{const.}$$

- CALCULATE THE BOUND CHARGE DENSITIES $\nearrow \rho_{S_b}$, $\searrow \rho_{V_b}$

WHEN WE HAVE A DIELECTRIC, THE VECTOR $\vec{D}$ IS EQUAL TO:

$$\vec{D} = \varepsilon \cdot \vec{E} = \varepsilon_0 \cdot \varepsilon_r \vec{E} = \vec{P} + \varepsilon_0 \vec{E}$$

$$\vec{P} = \varepsilon_0(\varepsilon_r - 1)\cdot \vec{E}$$

POLARIZATION VECTOR

THIS POL. VECTOR IS PROPORTIONAL TO $\vec{E}$ IN THE DIELECTRIC:

$$\vec{P} = \varepsilon_0(\varepsilon_r - 1)\cdot E(r)\Big|_{III} \cdot \vec{u}_r = \cancel{\varepsilon_0}(\varepsilon_r - 1)\cdot \overbrace{\left[\frac{Q}{4\pi\cancel{\varepsilon_0}\varepsilon_r}\cdot\frac{1}{r^2}\right]}^{\text{ELEC. FIELD IN } \boxed{III}} \vec{u}_r$$

THE BOUND DENSITY ARE DEFINED AS:

$$\begin{cases} \rho_{V_b} = -\nabla \cdot \vec{P} & (A) \\ \rho_{S_b} = P_n = \vec{P} \cdot \vec{u}_n & (B) \end{cases}$$

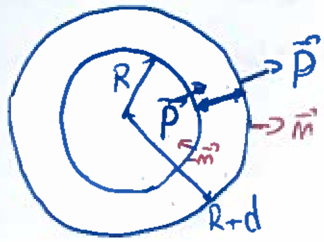
The DIELECTRIC IS AN ISOTROPIC MEDIUM

$\boxed{A}$ THE DIVERGENCE IN SPH. COORD IS

$$\nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot A_r\right) + \frac{\partial}{\partial\theta} \cdots + \frac{\partial}{\partial\phi}$$

$$\rho_{V_b} = -\nabla \cdot \vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left[\cancel{r^2}\cdot(\varepsilon_r - 1)\cdot Q \cdot \frac{1}{\cancel{r^2}}\right] = 0 \qquad \left(\text{perché } \frac{\partial}{\partial r}\varepsilon_r = 0\right)$$

$$\rho_{S_b} = \vec{P} \cdot \vec{u}_n$$



THE DIELECTRIC HAS A SURFACE WHOSE VECTOR NORMAL IS GOING AS $\vec{u}_r$, ...

$$\rho_{S_b} = \frac{\varepsilon_r - 1}{\varepsilon_r} \cdot \frac{Q}{4\pi r^2} \left(\vec{u}_r \cdot \vec{u}_n\right)$$

$\boxed{r = R}$ $\quad \rho_{S_b}(R) = \frac{\varepsilon_r - 1}{\varepsilon_r} \cdot \frac{Q}{4\pi R^2} \cdot (-1) = -\left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right) \cdot \frac{Q}{4\pi R^2}$

$\hookrightarrow \vec{u}_n \cdot \vec{u}_r = -1$

$\boxed{r = R + d}$ $\quad \rho_{S_b}(R+d) = \frac{(\varepsilon_r - 1)}{\varepsilon_r} \cdot \frac{Q}{4\pi (R+d)^2}$

$\hookrightarrow \vec{u}_n \cdot \vec{u}_r = 1$

TOTAL BOUND CHARGE :

$$Q_{bound} = \rho_{V_b} + \rho_{S_b}(R) \cdot S_R + \rho_{S_b}(R+d) \cdot S_{R+d} =$$

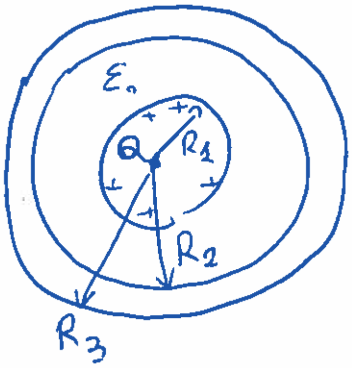
$$= \emptyset - \frac{(\varepsilon_r - 1)}{\varepsilon_r} \cdot \frac{Q}{\cancel{4\pi R^2}} \cancel{4\pi R^2} + \frac{(\varepsilon_r - 1)}{\varepsilon_r} \frac{Q}{\cancel{4\pi (R+d)^2}} \cdot \cancel{4\pi (R+d)^2}$$

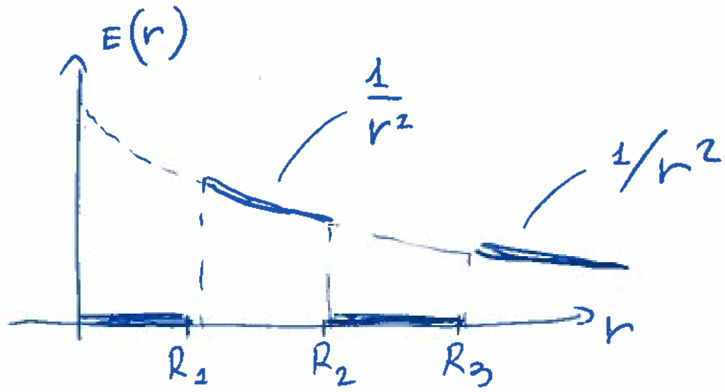
$$= \emptyset$$

EX 5

A CONDUCTIVE SPHERE WITH RADIUS R_1 AND CHARGE Q (>0) IS SURROUNDED BY A CONDUCTIVE SPHERICAL SHELL WITH INNER RADIUS R_2 AND OUTER RADIUS R_3.

- CALCULATE $\vec{E}$ AND V EVERYWHERE





$\vec{E}$ IS RADIAL (SYMMETRY)

4 REGIONS

I $E_r = 0$

III $E_r = 0$ → THIS MEANS THAT ON THE INNER SURFACE OF THE SHELL THERE MUST BE A SURFACE CHARGE DENSITY THAT IS CANCELLING OUT THE CONTRIBUTION OF Q: SO THERE IS A CHARGE $-Q$ SPREAD ON THE INNER SURFACE

$$\rho_S(R_2) = \frac{-Q}{S_2} = -\frac{Q}{4\pi R_2^2}$$

II GAUSS LAW

$$\int \vec{D}\cdot d\vec{s} = Q$$

$$\varepsilon_0 E_r \int ds = \boxed{\varepsilon_0 E(r) \cdot 4\pi r^2 = Q}$$

$$\boxed{E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

IV AS IN REGION II, THE ONLY CHARGE IS Q, SO:

$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}$$

(NO DIELECTRIC) ↓ VACUUM

SIMILARLY, ON THE OUTER SURFACE OF THE SECOND CONDUCTOR ~~...~~ IS INDUCED A POSITIVE CHARGE DENSITY:

$$Q_{TOT} = Q_{INT} - \cancel{S_2 \rho_{IND}} + \cancel{S_3 \rho_{IND}} = Q_{INT}$$

$$\rho_S(R_3) = \frac{+Q}{S_3} = \boxed{\frac{Q}{4\pi R_3^2}}$$

...NTIAL

IV $r > R_3$

$R_0 \to \infty$

$V(R_0) \to 0$

$$V(P) = -\int_{P_0}^{P} \vec{E} \cdot d\vec{e} + V(P_0) =$$

$$= -\int_{R_0}^{r} E_r \, dr + 0 = -\frac{Q}{4\pi\varepsilon_0}\int_{R_0}^{r} \frac{1}{r^2} dr =$$

$$= \boxed{\frac{Q}{4\pi\varepsilon_0}\frac{1}{r}}$$

$$V(R_3) = \frac{Q}{4\pi\varepsilon_0 R_3}$$

III $R_2 < r < R_3$

$$V(r) = -\int_{R_3}^{r} \underbrace{E_r}_{0} \, dr + V(R_3) = V(R_3) = \text{const}$$

II $R_1 < r < R_2$

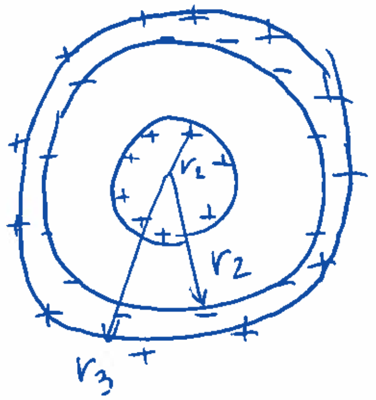
$$V(r) = -\int_{R_2}^{r} E_r\big|_{II} \, dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$V(R_1) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{1}{R_3}\right)$$

I

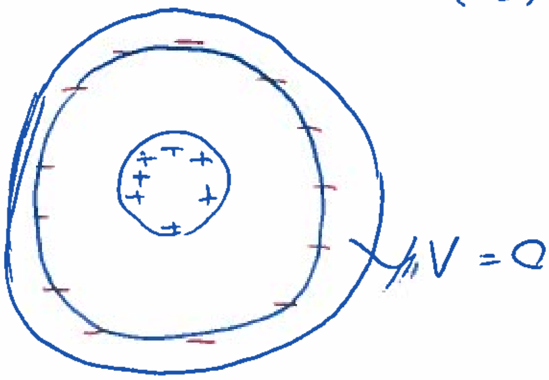
$$V(r) = V(R_1)$$

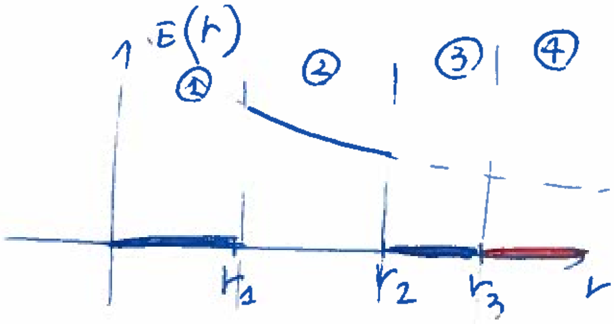
Q: WHAT HAPPENS IF THE OUTER CONDUCTIVE SPHERE IS GROUNDED?

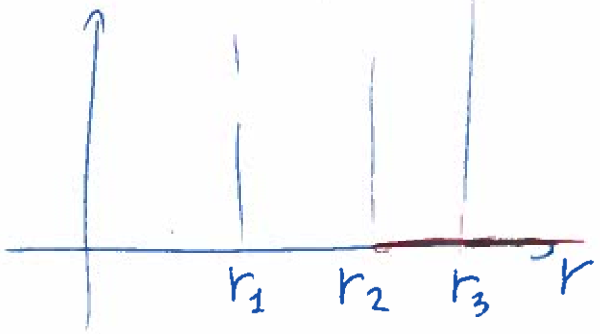


IN OUR ORIGINAL SITUATION, THE EXTERNAL CONDUCTOR WAS INFLUENCED BY THE INNER ONE, SO ~~IT~~ WAS PRODUCING A FIELD TO REACT AT THE EXTERNAL ONE

IF WE GROUND THE SECOND CONDUCTOR, IT MEANS THAT THE POTENTIAL $V(R_3)=0$ IN THE CONDUCTOR:







ELECTRIC FIELD

1-3 NOTHING CHANGES: WE CAN CONTINUE TO APPLY GAUSS LAW

4 NOW WE HAVE THAT THE CHARGES PREVIOUSLY SPREADED OVER R_3 MOVE TO THE GROUND:

$$\rho(R_3)=0 \longrightarrow$$

NOW IF WE PUT A sphere WITH RADIUS $r > R_3$ THE TOTAL CHARGE Q IS ZERO

$\downarrow$

$E=0$

POTENTIAL

3-4 FORCED TO ZERO AND CONSTANT

2 $$V(r) = -\int_{r_2}^{r} \vec{E}\cdot d\vec{r} + \cancel{V(r_2)} = -E_r\Big|_{II}\cdot(r-r_2) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{R_2}\right)$$

1 $$V(r)=V(R_1)=\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$