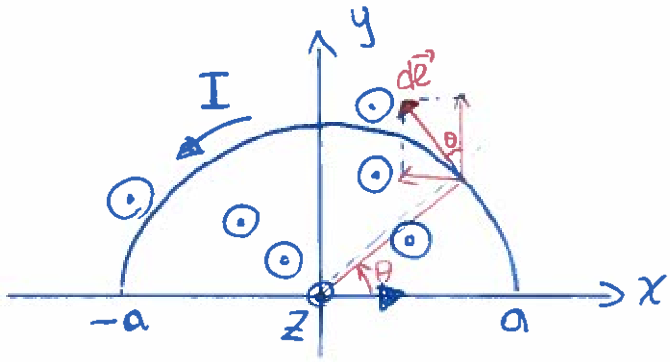
EX 1 (AMPERE FORCE)

A CIRCUIT CARRYING A CURRENT I IS PLACED IN A REGION WITH A UNIFORM MAGNETIC FLUX DENSITY $\vec{B}$, PERPEDICULAR TO THE CIRCUIT (SEMICIRCLE WITH RADIUS a).

CALCULATE THE MAGNETIC FORCE ACTING ON THE CIRCUIT.



$$\vec{B} = B_0 \vec{u}_z$$

WE KNOW THAT THE FORCE GENERATED BY THE MAGNETIC FLUX DENSITY $\vec{B}$ ON A PIECE OF CIRCUIT $d\vec{\ell}$ THAT IS CARRYING CURRENT I IS :

$$d\vec{F} = I\, d\vec{\ell} \times \vec{B} \quad \text{(AMPERE FORCE)}$$

- $d\vec{\ell}$ IS DIRECTION IN WHICH CURRENT IS FLOWING
- CROSS PRODUCT : $d\vec{F}$ LAYS IN xy PLANE

INTEGRATING OVER THE ENTIRE CIRCUIT WE FIND THE FORCE :

$$\vec{F} = \int_C I\, d\vec{\ell} \times \vec{B} = \underbrace{\int_{\text{STRAIGHT LINE}} I\, d\vec{\ell} \times \vec{B}}_{\text{I}} + \underbrace{\int_{\text{SEMI CIRCLE}} I\, d\vec{\ell} \times \vec{B}}_{\text{II}}$$

I $\left.\begin{array}{l} d\vec{\ell} = dx\, \vec{u}_x \\ \vec{B} = B_0 \vec{u}_z \end{array}\right\} \quad d\vec{\ell} \times \vec{B} = dx\, B_0 (-\vec{u}_y)$

$$\int_{\text{S.LINE}} I\, d\vec{\ell} \times \vec{B} = \int_{x=-a}^{a} I B_0 (-\vec{u}_y)\, dx = I \cdot B_0 (-u_y) \left[x \right]_{-a}^{a} = \boxed{(-2 I B_0 a) \vec{u}_y}$$

II $d\vec{\ell} = (a\, d\theta)\, \vec{u}_\theta$

WE DECOMPOSE THIS UNIT VECTOR IN CARTESIAN COORDINATES

$$\vec{u}_\theta = \sin\theta\, (-\vec{u}_x) + \cos\theta\, (\vec{u}_y)$$

$$\int_{\substack{\text{semi}\\\text{circle}}} I\,d\vec{\ell} \times \vec{B} = \int_{\theta=0}^{\pi} I a \cdot B_0 \left\{ \left(\overbrace{\sin\theta(-\vec{u}_x) \times \vec{u}_z}^{u_y} \right) + \left(\overbrace{\cos\theta\, \vec{u}_y \times \vec{u}_z}^{\vec{u}_x} \right) \right\} d\theta =$$

$$= I a B_0 \vec{u}_y \int_0^{\pi} \sin\theta\, d\theta + I a B_0 \vec{u}_x \int_0^{\pi} \cos\theta\, d\theta =$$

$$= I a B_0 \vec{u}_y \left[-\cos\theta\right]_0^{\pi} + \cancel{I a B_0 \vec{u}_x \left[\sin\theta\right]_0^{\pi}} =$$

$$= I \cdot a B_0 \vec{u}_y \left[+1 \; +1\right] = \boxed{+2 I a B_0 \vec{u}_y}$$

SUMMING UP THE TWO CONTRIBUTIONS

$$\vec{F} = \vec{F}_{\text{straight}} + \vec{F}_{\substack{\text{semi}\\\text{circle}}} = -2 I B_0 a\, \vec{u}_y + 2 I a B_0 \vec{u}_y = \emptyset$$

COMMENT : THIS RESULT IS NOT SURPRISING AND HOLDS FOR ANY CIRCUIT SHAPE THAT IS IMMERSED IN A MAGNETIC FLUX DENSITY
→ UNIFORM
→ ORTHOGONAL TO THE CIRCUIT.

INFACT :

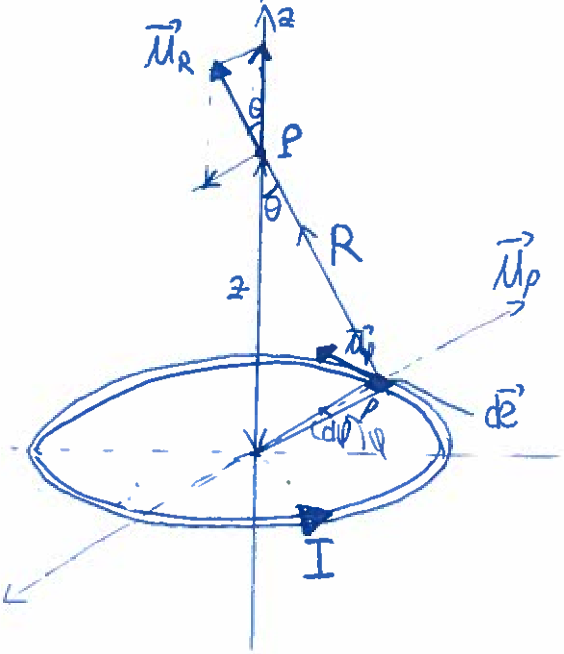
$$\vec{F} = \oint_C I\,d\vec{\ell} \times \vec{B} = I \oint_C d\vec{\ell} \times \vec{B} =$$

$$= I \underbrace{\left(\oint_C d\vec{\ell} \right)}_{\emptyset} \times \vec{B} = \emptyset$$

if $\vec{B}$ IS UNIFORM AND ORTHOGONAL TO THE CIRCUIT

EX 2 | (BIOT-SAVART LAW)

CALCULATE THE MAGNETIC FLUX DENSITY $\vec{B}$ PRODUCED BY A CIRCULAR LOOP (WITH RADIUS ρ) CARRYING A CURRENT I, AT A POINT ALONG ITS AXIS.



BIOT - SAVART LAW

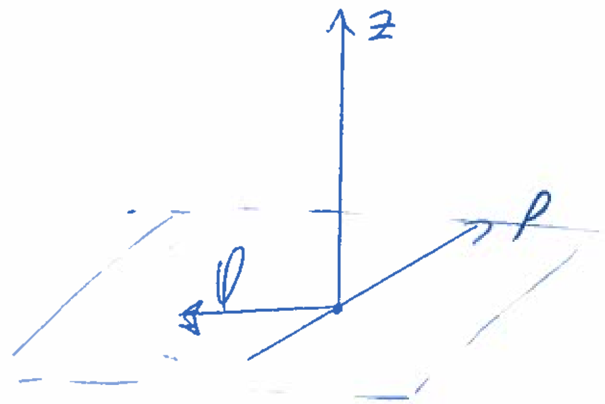
$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\vec{l} \times \vec{u}_R}{R^2}$$

THE LAW DESCRIBE THE ELEMENTAR MAGNETIC FLUX DENSITY GENERATED BY AN ELEMENTAR ELEMENT $d\vec{l}$ THAT IS CARRYING CURRENT I

$$\boxed{d\vec{l} = \rho \, d\varphi \, \vec{u}_\varphi}$$

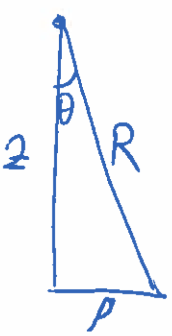
CIRCUIT ELEMENT HAS THE DIRECTION OF CURRENT IN IT

IN CYLINDRICAL COORDINATES we have:



$\vec{u}_R$ CAN BE DECOMPOSED IN TWO COMPONENTS ALONG $\vec{u}_z$ AND $\vec{u}_\varphi$:

$$\boxed{\vec{u}_R = \cos\theta \, \vec{u}_z + \sin\theta \left(-\vec{u}_\rho\right)}$$



$$R = \sqrt{z^2 + \rho^2}$$

$$\sin\theta = \frac{\rho}{R}$$

$$\cos\theta = \frac{z}{R}$$

WE HAVE ALL THE ELEMENT THAT WE NEED TO FIND $d\vec{B}$

$$d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{I}{R^2}\left(d\vec{\ell}\times\vec{u}_R\right) =$$

$$= \frac{\mu_0\cdot I}{4\pi R^2}\left[\rho\, d\varphi\, \vec{u}_\varphi \times \left(\cos\theta\,\vec{u}_z - \sin\theta\,\vec{u}_\rho\right)\right] =$$

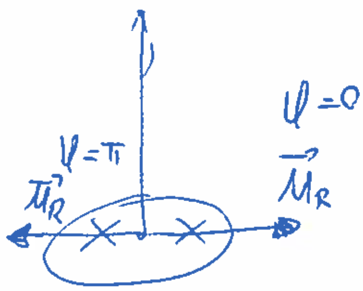
$$= \frac{\mu_0 I}{4\pi R^2}\left[\rho\, d\varphi\cos\theta\,\overbrace{\left(\vec{u}_\varphi\times\vec{u}_z\right)}^{\vec{u}_\rho} - \rho\, d\varphi\sin\theta\,\overbrace{\left(\vec{u}_\varphi\times\vec{u}_\rho\right)}^{-\vec{u}_z}\right] =$$

$$= \frac{\mu_0 I}{4\pi R^2}\left[\rho\cos\theta\, d\varphi\,\vec{u}_\rho + \rho\sin\theta\, d\varphi\,\vec{u}_z\right]$$

I INTEGRATE ALONG THE CIRCUIT TO GET $\vec{B}$:

$$\vec{B} = \oint_C d\vec{B} = \int_0^{2\pi}\frac{\mu_0 I}{4\pi R^2}\left[\frac{\rho z}{R}\,\vec{u}_\rho\, d\varphi + \frac{\rho^2}{R}\,\vec{u}_z\, d\varphi\right] =$$

$$= \frac{\mu_0 I}{4\pi R^3}\left[\rho z\cdot\underbrace{\int_0^{2\pi}\vec{u}_\rho\, d\varphi}_{=0} + \rho^2\int_0^{2\pi}\vec{u}_z\, d\varphi\right] =$$

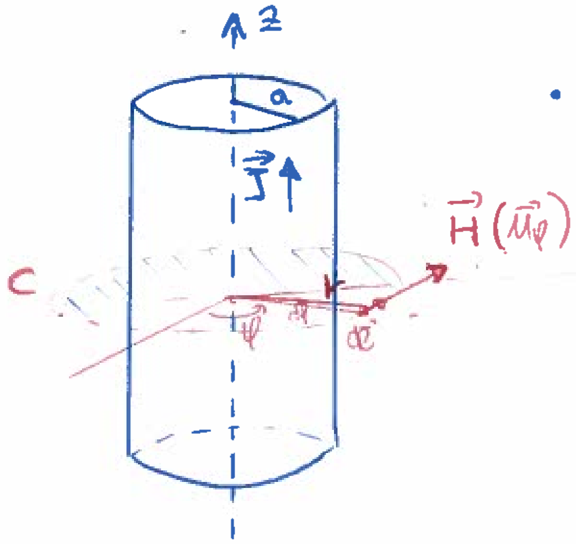


$$= \frac{\mu_0 I}{4\pi R^3}\cdot\rho^2\cdot 2\pi\,\vec{u}_z =$$

$$= \frac{\mu_0 I\cdot\rho^2\cdot\cancel{2\pi}}{\cancel{4\pi}_{2}\left(z^2+\rho^2\right)^{3/2}}\,\vec{u}_z = \boxed{\frac{\mu_0}{2}\cdot I\cdot\frac{\rho^2}{\left(z^2+\rho^2\right)^{3/2}}\,\vec{u}_z}$$

EX 3 (AMPERE LAW)

CALCULATE THE MAGNETIC FIELD $\vec{H}$ GENERATED BY AN INFINITELY EXTENDED CYLINDER WITH RADIUS a, CARRYING AN UNIFORM CURRENT I (CURRENT DENSITY IS $\vec{J}$)



- THE CURRENT DENSITY IS A VECTOR THAT GOES IN DIRECTION OF THE CURRENT IS EQUAL TO THE TOTAL CURRENT DIVIDED BY THE AREA OF THE SECTION THROUGH WHICH I IS FLOWING:

$$\vec{J} = \frac{I}{\pi a^2}\vec{u}_z$$

ⓐ• DUE TO THE SIMMETRY OF THE Problem, THE MAGNETIC FIELD DOESN'T DEPEND ON z AND φ, BUT JUST ON r.

ⓑ• KEEP IT MIND THAT THIS DOESN'T MEAN THAT $\vec{H}$ IS DIRECTED AS $\vec{u}_r$! WE KNOW THAT THE "FIELD LINES" OF THE MAGNETIC FIELD ARE RING CLOSED ON THE WIRE WITH CURRENT.

→ SO $\boxed{\vec{H} = H_\varphi \vec{u}_\varphi = H(r)\vec{u}_\varphi}$

NOW WE CAN APPLY AMPERE'S LAW:

$$\oint_C \vec{H}\cdot d\vec{\ell} = \int_S \vec{J}\cdot d\vec{S}$$

THE LAW TELLS THAT THE INTEGRAL OF MAGNETIC FIELD ALONG A CLOSED LINE IS EQUAL TO THE FLUX OF DENSITY CURRENT THROUGH A SURFACE (IDENTIFIED BY THE CLOSED LINE)

1ST PART OF INTEGRAL:

$\vec{H} = H_\varphi \vec{u}_\varphi$

$d\vec{\ell} = d\ell\,\vec{u}_\varphi = \boxed{r\cdot d\varphi\,\vec{u}_\varphi}$

$$\oint_C \vec{H}\cdot d\vec{\ell} = \int_C H_\varphi\, d\ell = H_\varphi\int r\,d\varphi = H_\varphi\cdot r\int_0^{2\pi} d\varphi = \boxed{2\pi r\cdot H_\varphi}$$

WE CHOOSE A CIRCUMF. WITH RADIUS r: WE KNOW H_φ IS CONSTANT ALONG A CIRC.

(WE DRAW A close line → WE SET AN INTEGRATION PATH → FOR MAG. FIELD

2^{nd} PART OF INTEGRAL

$$\int_S \vec{J}\cdot d\vec{s} \qquad \vec{J} = J\,\vec{u}_z \qquad d\vec{s} = ds\,\vec{u}_z$$

WE NEED TO DISTINGUISH TWO CASES:

↳ if $\boxed{r<a}$ (INSIDE CYLINDER)

$$\int_S \vec{J}\cdot d\vec{s} = \left(\frac{I}{\pi a^2}\right)\cdot \pi r^2 = I\left(\frac{r}{a}\right)^2$$

SO: $2\pi r\cdot H_\varphi = I\left(\frac{r}{a}\right)^2$

$$H_\varphi = \frac{I}{2\pi}\cdot\frac{r}{a^2}$$

$$\downarrow$$

$$\boxed{\vec{H} = \left(\frac{I}{2\pi}\cdot\frac{r}{a^2}\right)\vec{u}_\varphi}$$

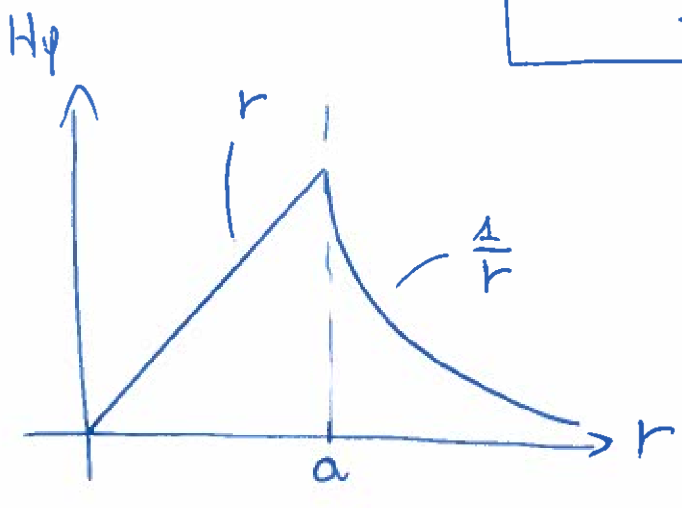
↳ if $r \geq a$

$$\int_S \vec{J}\cdot d\vec{s} = \frac{I}{\cancel{\pi a^2}}\cdot\cancel{\pi a^2} = I$$

SO: $2\pi r\cdot H_\varphi = I$

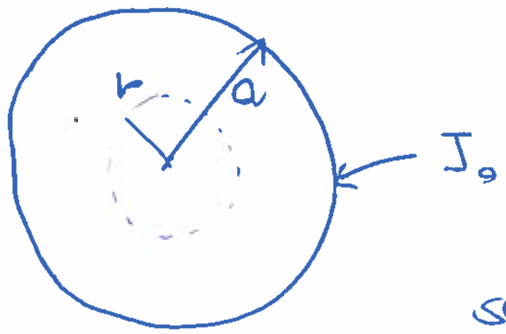
$$H_\varphi = \frac{I}{2\pi r}$$

$$\boxed{\vec{H} = \frac{I}{2\pi r}\,\vec{u}_\varphi}$$



ACTUALLY, A CONDUCTOR CAN SUPPORT A UNIFORM CURRENT DENSITY (as in the exercise) ONLY WHEN IT CARRIES A DIRECT CURRENT (D.C.).

WHEN CURRENT IS TIME DEPENDENT $\left(\frac{di}{dt} \neq 0,\ \text{AC REGIME}\right)$ THE CURRENT IS CONFINED NEAR THE OUTER REGION OF THE CONDUCTOR : $\boxed{\text{SKIN EFFECT}}$



$J(a) = J_0$ CURRENT DENSITY AT THE OUTER SURFACE

SO, IN A <u>REAL</u> CYLINDRICAL CONDUCTOR, THE CURRENT DENSITY CAN BE DESCRIBED AS:

$$J(r) = J_0\, e^{-\frac{1}{\delta}(a-r)}$$

where δ is the <u>SKIN DEPTH</u>

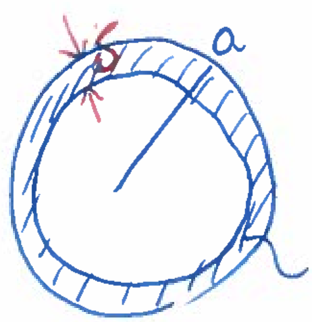
IF THE TOTAL CURRENT CARRIED BY THE CONDUCTOR IS I_0, WE HAVE:

$$I_0 = \int \vec{J}(r)\cdot d\vec{s} = \int_0^{2\pi} d\theta \int_0^a r \cdot J(r)\, dr = 2\pi J_0 \int_0^a r \cdot e^{-\frac{1}{\delta}(a-r)}\, dr$$

$$= \boxed{2\pi J_0\, a\, \delta}$$

if $a \gg \delta$

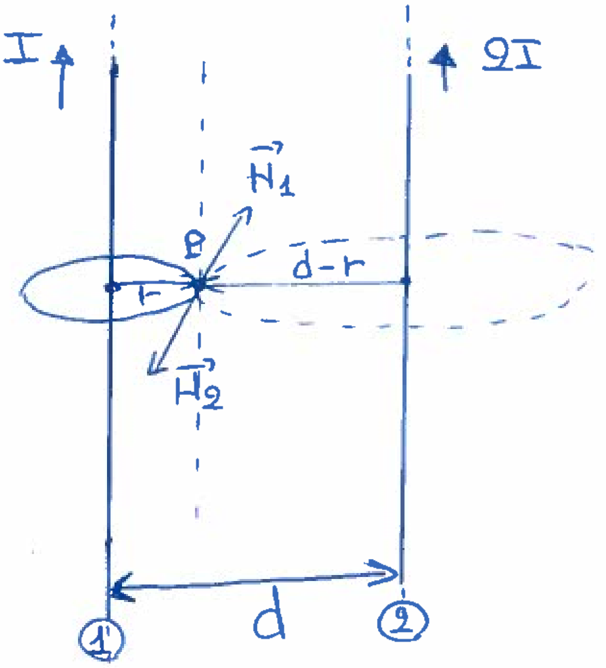
SO IS LIKE THE TOTAL CURRENT I_0 FLOWING IN A REAL CONDUCTOR IS GIVEN BY THE PRODUCT OF THE CURRENT DENSITY J_0 AND THE AREA OF A CIRCULAR SECTOR WITH A WIDTH EQUAL TO THE SKIN DEPTH δ.



$$I = J_0 (2\pi a \cdot \delta)$$

EX 4 (AMPERE LAW)

TWO INFINITELY LONG AND STRAIGHT WIRES ARE PARALLEL AND PLACED AT A DISTANCE d TO EACH OTHER.
THE FIRST WIRE CARRIES A CURRENT I, WHILE THE SECOND ONE CARRIES A CURRENT $2I$ (FLOWING IN THE SAME DIRECTION).
FIND THE POINTS where THE TOTAL MAGNETIC FIELD $\vec{H}$ VANISHES.



$(d = 60 \text{ cm})$

$H(P) = 0$?

AGAIN, FROM GEOMETRIC REASONS:

$$\vec{H}_{1/2} = H_\varphi(r)\,\vec{u}_\varphi$$

WE APPLY AMPERE'S LAW TO BOTH WIRES $\oint_C \vec{H}\cdot d\vec{\ell} = \int_S \vec{J}\cdot d\vec{S}$

Wire 1 $\int_C H_{1\varphi}\cdot r\cdot d\theta = H_1\cdot r\int_0^{2\pi} d\theta = H_1\cdot 2\pi r = I \longrightarrow H_1 = \frac{I}{2\pi r}$

Wire 2 $\int_C H_{2\varphi}\cdot(d-r)\,d\theta = H_2(d-r)\cdot 2\pi = 2I \longrightarrow H_2 = \frac{2I}{2\pi(d-r)}$

$$\vec{H}(P) = \vec{H}_1(P) + \vec{H}_2(P) = |\vec{H}_1(P)|\,\vec{u}_\varphi + |\vec{H}_2(P)|(-\vec{u}_\varphi) =$$

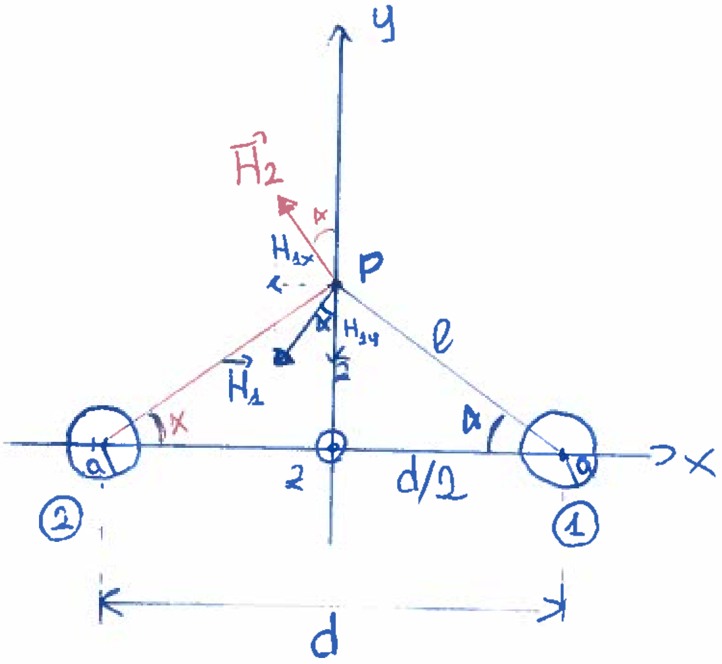
$$= \underbrace{\{|\vec{H}_1(P)| - |\vec{H}_2(P)|\}}_{=0}\,\vec{u}_\varphi = 0$$

$$H_1 + H_2 = \frac{I}{2\pi r} + \frac{2I}{2\pi(d-r)} = \frac{I(d-r)+2Ir}{2\pi(d-r)\cdot r} = 0 \rightarrow \boxed{r = \frac{d}{3}} = \boxed{20\text{ cm}}$$

EX 5 (AMPERE LAW)

TWO INFINITELY LONG CYLINDRICAL CONDUCTORS WITH RADIUS a ARE PARALLEL AND DISTANT TO EACH OTHER d (see figure). BOTH CYLINDERS CARRY A CURRENT DENSITY $\vec{J}$, DIRECT ALONG z-AXIS.

CALCULATE THE TOTAL MAGNETIC FIELD GENERATED IN POINT $P(0, z)$.



$$\ell = \sqrt{z^2 + \left(\frac{d}{2}\right)^2}$$

$$\alpha = \arctan\left(\frac{z}{d/2}\right)$$

REGARDING THE <u>FIRST</u> CONDUCTOR 1 WE APPLY AMPERE LAW:

$$\underbrace{\oint_c \vec{H} \cdot d\vec{\ell}}\ = \int_S \vec{J} \cdot d\vec{s}$$

$$H_{1\varphi} \cdot 2\pi\ell = J_0 \pi a^2$$

$$\vec{H}_1 = H_{1\varphi} \vec{u}_\varphi \qquad d\vec{s} = ds\, \vec{u}_z$$

$$d\vec{\ell} = \ell\, d\varphi\, \vec{u}_\varphi \qquad \vec{J} = J_0 \vec{u}_z$$

$$H_{1\varphi} = \frac{J_0 a^2}{2\ell}$$

$$\vec{H}_1 = H_{1\varphi} \vec{u}_\varphi = (-H_{1\varphi} \cdot \cos\alpha)\vec{u}_y - (H_{1\varphi} \cdot \sin\alpha)\vec{u}_x$$

ABOUT <u>SECOND</u> CONDUCTOR 2:

$$\underbrace{H_{2\varphi} = H_{1\varphi}}$$

$$\vec{H}_2 = (H_{1\varphi} \cdot \cos\alpha\, \vec{u}_y) - (H_{1\varphi} \cdot \sin\alpha)\vec{u}_x$$

SAME MODULUS → ≠ DIRECTION

THE TOTAL MAGNETIC FIELD $\vec{H}$:

$$\vec{H} = \vec{H}_1 + \vec{H}_2 = \left(-2H_{1\varphi}\cdot\sin\alpha\right)\vec{u}_x = \boxed{-2\sin\alpha\,\frac{J_0\,a^2}{2\sqrt{z^2+\left(\frac{d}{2}\right)^2}}\;\vec{u}_x}$$

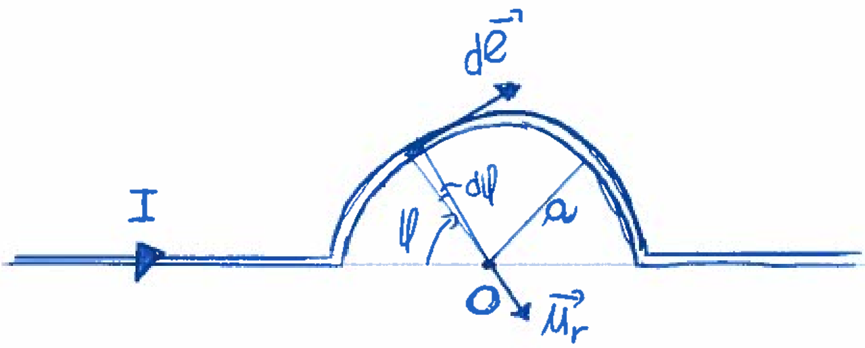
if

$$\left|\begin{array}{l} J_0 = 6366{,}2\left[\dfrac{A}{m^2}\right] \\ \\ a = 1\text{ cm} \\ \\ d = 5\text{ cm} \\ \\ z = 3\text{ cm} \end{array}\right| \longrightarrow \vec{H} = \left(-12{,}56\right)\vec{u}_x\;\left[\frac{A}{m}\right]$$

EX (BIOT SAVART LAW)

CONSIDER THE BENT WIRE (WITH RADIUS a) SHOWN IN THE FIGURE, CARRYING A CURRENT I.

CALCULATE THE MAGNETIC FIELD FLUX DENSITY $\vec{B}$ PRODUCED IN THE POINT O.



THE UNIT-VECTOR $\vec{u}_z$ IS OUTGOING FROM THE PLANE.

AS USUAL WE APPLY THE BIOT-SAVART LAW, TAKEN FROM AMPERE FORCE

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \vec{u}_R}{R^2}$$

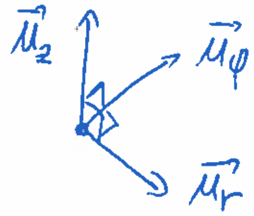
- $d\vec{\ell}$ IS THE DIFFERENTIAL PATH LENGTH IN WHICH THE CURRENT IS FLOWING
- $\vec{u}_R$ IS THE UNIT VERSOR THAT GOES FROM THE POINT $d\vec{\ell}$ TO THE ONE IN WHICH WE NEED TO FIND $\vec{B}$

IN THE HALF CIRCLE

$$d\vec{\ell} = (a \cdot d\varphi)\vec{u}_\varphi$$

$$R = a$$

$$\vec{u}_R = \vec{u}_r$$



THE 3 VECTORS ARE ORTHOGONAL

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I}{a^2}\Big[(a\, d\varphi)\underbrace{(\vec{u}_\varphi \times \vec{u}_r)}_{-\vec{u}_z}\Big] =$$

$$= -\frac{\mu_0 I \cancel{a}}{4\pi a^{\cancel{2}}}\, d\varphi\, \vec{u}_z$$

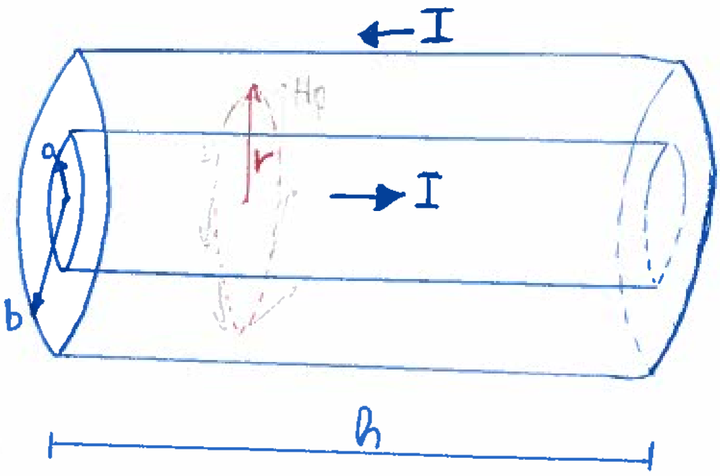
$$\vec{B} = \int_{\varphi=0}^{\pi} d\vec{B} = \int_0^{\pi} -\frac{\mu_0 I}{4\pi a}\, d\varphi\, \vec{u}_z = -\frac{\mu_0 I \cancel{\pi}}{4\cancel{\pi} a}\vec{u}_z = \boxed{-\frac{\mu_0 I}{4a}\vec{u}_z}$$

$\vec{B}$ IS IN-GOING INTO THE PLANE

COMMENT: THE CONTRIBUTION OF THE STRAIGHT WIRES TO $\vec{B}$ IS ZERO, BECAUSE IN THAT CASE $d\vec{\ell} \times \vec{u}_R = 0$ (GIVEN THAT $d\vec{\ell} \,//\, \vec{u}_R$)

EX 7 (INDUCTANCE)

CALCULATE THE INDUCTANCE PER UNIT LENGTH (L_ℓ) OF A COAXIAL CABLE WITH INNER RADIUS a AND OUTER RADIUS b.



THE CURRENT THAT IS FLOWING IN THE INNER CYLINDRICAL CONDUCTOR IS EQUAL TO THE COUNTER-PROPAGATING CURRENT CARRIED BY THE EXTERNAL CONDUCTOR

APPLYING THE AMPERE LAW : $\oint_C \vec{H} \cdot d\vec{\ell} = \int_S \vec{J} \cdot d\vec{S}$

- $\boxed{r > b}$ $\int_S \vec{J} \cdot d\vec{S} = I - I = \emptyset$

  ↓

  $\vec{H}$ IS ZERO OUTSIDE OF COAXIAL CABLE

  THE CURRENT IN THE INNER CONDUCTOR IS CANCELLED OUT BY THE CURRENT IN THE OUTER

- $\boxed{a < r < b}$ DUE TO THE SYMMETRY WE KNOW THAT $\vec{H} = H_\varphi \vec{u_\varphi}$

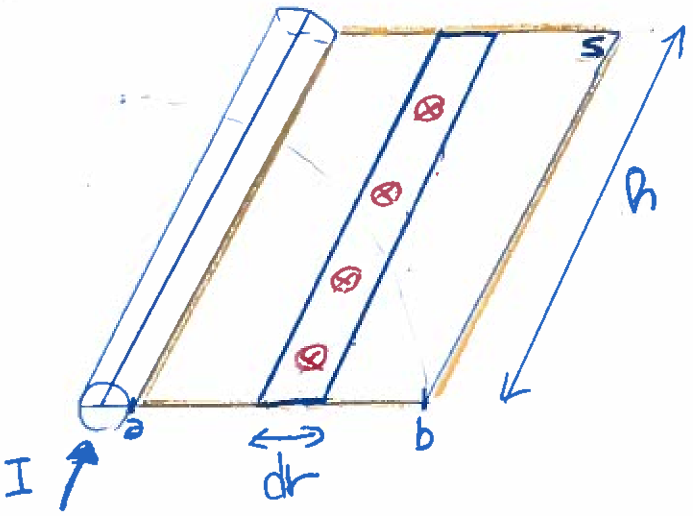
$$\oint_C \vec{H} \cdot d\vec{\ell} = \int_C (H_\varphi \vec{u_\varphi}) \cdot (r \cdot d\varphi \, \vec{u_\varphi}) = H_\varphi r \int_0^{2\pi} d\varphi = 2\pi \cdot r \cdot H_\varphi$$

$$\int_S \vec{J} \cdot d\vec{S} = I \longrightarrow H_\varphi = \frac{I}{2\pi r} \longrightarrow B_\varphi = \mu H_\varphi = \frac{\mu I}{2\pi r}$$

THE SELF-INDUCTANCE L IS DEFINED AS THE RATIO BETWEEN THE FLUX OF $\vec{B}$ OVER THE SURFACE BETWEEN THE CONDUCTORS AND THE CURRENT FLOWING IN IT.

$$L = \frac{\phi_m}{I} = \frac{\int_S \vec{B} \cdot d\vec{S}}{I}$$

WE SEE THE COAXIAL FROM THIS PERSPECTIVE.



$$L = \frac{\int_S \vec{B} \cdot d\vec{S}}{I}$$

$\vec{B}$ IS ENTERING IN THIS SURFACE ELEMENT $d\vec{S}$: $\vec{B} = \frac{\mu I}{2\pi r} \vec{u}_\varphi$

: $d\vec{S} = dS \, \vec{u}_\varphi = (dr \cdot h) \, \vec{u}_\varphi$

$$\Phi_m = \int_S \vec{B} \cdot d\vec{S} = \int_a^b \frac{\mu I}{2\pi r} dr \cdot h =$$

$$= \int_a^b \left(\frac{\mu I \cdot h}{2\pi} \right) \frac{dr}{r} = \frac{\mu I h}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$L = \frac{\Phi_m}{I} = \frac{\mu_0 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

DIVIDING BY THE COAXIAL LENGTH h WE FIND:

$$L_\ell = \frac{L}{h} = \boxed{\frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right)}$$

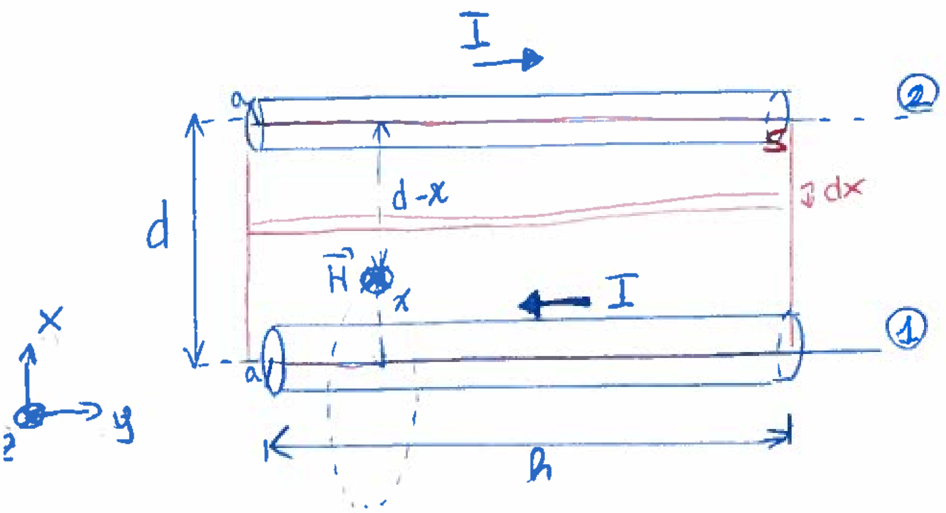
COMMENT: REMEMBERING THAT THE CAPACITANCE P.U.L. OF COAXIAL CABLE WAS $C_\ell = \frac{2\varepsilon\pi}{\ln(b/a)}$ WE GET:

$$\boxed{L_\ell \cdot C_\ell = \varepsilon \cdot \mu}$$

THIS RELATION HOLDS FOR ANY TRANSMISSION LINE IN A HOMOGENEOUS MEDIUM

EX 1 (INDUCTANCE)

CALCULATE THE INDUCTANCE PER UNIT LENGTH (L_e) OF A BIFILAR (TWO-WIRE) LINE. THE TWO WIRES HAVE RADIUS a AND ARE PLACED AT DISTANCE d.



THE TOTAL CURRENT FLOWING IN THE TWO WIRES (IN OPPOSITE DIRECTION) IS I.

LET'S CONSIDER A BIFILAR LINE WITH A LENGTH h AND CALCULATE ITS SELF INDUCTANCE L:

$$L = \frac{\Phi_m}{I} = \frac{\int_S \vec{B} \cdot d\vec{s}}{I}$$

SO FIRST OF ALL WE NEED TO CALCULATE THE MAGNETIC FLUX DENSITY $\vec{B}$ GENERATED IN REGION BETWEEN THE TWO CONDUCTORS. WE APPLY AMPERE LAW USING THE SUPERIMPOSITION OF EFFECTS (① + ②), EVALUATING $\vec{H}$ IN A POINT PLACED AT DISTANCE x FROM FIRST CONDUCTOR.

$$\oint_C \vec{H} \cdot d\vec{e} = \int_S \vec{J} \cdot d\vec{s}$$

FOR SYMMETRY REASON $\vec{H} = H_\varphi(r)\,\vec{u}_\varphi$

① $\oint_C (+H_1 \vec{u}_z) \cdot (d\varphi \cdot x\,\vec{u}_z) = I$ $\qquad +H_1 \cdot x \cdot 2\pi = I$ $\qquad H_1 = +\dfrac{I}{2\pi x}$

② $\oint_C (H_2 \vec{u}_z) \cdot (d\varphi (d-x)\,\vec{u}_z) = I$ $\qquad H_2 (d-x) 2\pi = I$ $\qquad H_2 = \dfrac{I}{2\pi(d-x)}$

$$\vec{H} = \vec{H}_1 + \vec{H}_2 = \left[+ \frac{I}{2\pi x} + \frac{I}{2\pi(d-x)} \right] \vec{u}_z$$

"THE TOTAL MAGNETIC FIELD IS ENTERING IN THE SHEET"

$$\vec{B} = \mu \cdot \vec{H} = \left[\frac{+\mu I}{2\pi x} + \frac{\mu I}{2\pi(d-x)} \right] \vec{u}_z$$

THE FLUX OF $\vec{B}$ OVER SURFACE S IS:

$$\phi_m = \int_S \vec{B} \cdot d\vec{s} = \int_{x=a}^{d-a} \left[\frac{+\mu I}{2\pi x} + \frac{\mu I}{2\pi(d-x)} \right] \cdot h\, dx =$$

$$d\vec{s} = ds\, \vec{u}_z = h \cdot dx\, \vec{u}_z$$

$$= + \frac{\mu I \cdot h}{2\pi} \int_a^{d-a} \left(\frac{dx}{x} \right) + \frac{\mu I h}{2\pi} \int_a^{d-a} \frac{dx}{d-x} =$$

$$= \frac{\mu I h}{2\pi} \left[\ln x \right]_a^{d-a} - \frac{\mu I h}{2\pi} \left[\ln(d-x) \right]_a^{d-a} = \frac{\mu I h}{2\pi} \left[\left(\ln(x) \right)_a^{d-a} - \left(\ln(d-x) \right)_a^{d-a} \right] =$$

$$= \frac{\mu I h}{2\pi} \left[\ln(d-a) - \ln(a) - \ln(\cancel{d} - \cancel{d} + a) + \ln(d-a) \right] =$$

$$= \frac{\mu I h}{2\pi} \left[2 \ln(d-a) - 2 \ln(a) \right] = \frac{2 \mu I h}{2\pi} \left[\ln \left(\frac{d-a}{a} \right) \right] \simeq \quad d \gg a$$

$$= \frac{\mu I h}{\pi} \ln \left(\frac{d}{a} \right)$$

$$L = \frac{\phi_m}{I} = \frac{\mu h}{\pi} \ln \left(\frac{d}{a} \right) \longrightarrow L_e = \frac{L}{h} = \boxed{\frac{\mu}{\pi} \ln \left(\frac{d}{a} \right)}$$

COMMENT: WE KNOW THAT IN A <u>HOMOGENEOUS MEDIUM</u> $C_e L_e = \mu \varepsilon$. SO WE FIND THE CAPACITANCE P.U.L. OF A BIFILAR LINE:

$$C_e = \frac{\mu \varepsilon}{L_e} = \frac{\cancel{\mu} \varepsilon}{\frac{\cancel{\mu}}{\pi} \ln \left(\frac{d}{a} \right)} = \boxed{\frac{\varepsilon \cdot \pi}{\ln \left(\frac{d}{a} \right)}} \longrightarrow$$

RESULT ALREADY FOUND USING CAPACITANCE DEFINITION

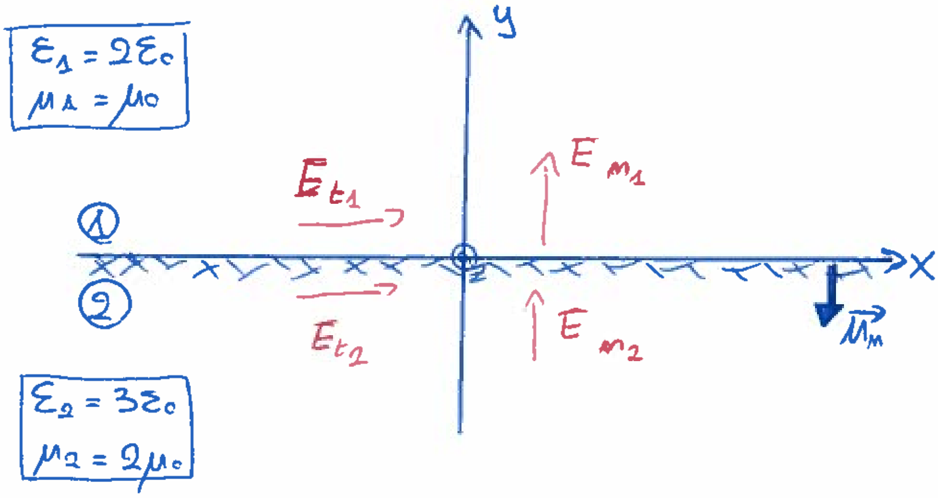
EX 2 (BOUNDARY CONDITIONS)

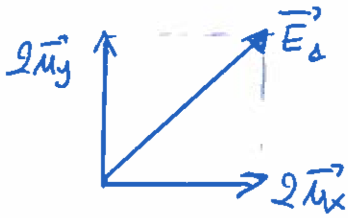
THE $\boxed{\text{PLANE } y=0}$ IS THE BOUNDARY INTERFACE BETWEEN TWO MATERIALS (1 AND 2) WITH PARAMETERS ε_1, μ_1 AND ε_2, μ_2.

THE ELECTRIC FIELD IN 1 IS $\vec{E_1} = 2(\vec{u_x} + \vec{u_y})$ [V/m]

THE MAGNETIC FIELD IN 2 IS $\vec{H_2} = 10\,\vec{u_x} + \vec{u_y}$ [A/m]

ASSUMING NO CHARGES AND NO CURRENT DENSITY ON THE INTERFACE CALCULATE $\vec{E_2}$ AND $\vec{H_1}$.





N.B. PLANE $y=0$ MEANS THE x-z PLANE!

THE BOUNDARY CONDITIONS ARE:

ELECTRIC FIELD
$$(\vec{D_2} - \vec{D_1}) \cdot \vec{u_n} = \cancel{\rho_s}$$
$$(\vec{E_2} - \vec{E_1}) \times \vec{u_n} = 0$$

MAGNETIC FIELD
$$(\vec{H_2} - \vec{H_1}) \times \vec{u_n} = \cancel{-\vec{J_s}}$$
$$(\vec{B_2} - \vec{B_1}) \cdot \vec{u_n} = 0$$

$\rightarrow$
$$\boxed{\begin{aligned} D_{n2} - D_{n1} &= 0 \\ E_{t2} - E_{t1} &= 0 \\ H_{t2} - H_{t1} &= 0 \\ B_{n2} - B_{n1} &= 0 \end{aligned}}$$

TANGENT COMPONENTS OF FIELD

NORMAL COMPONENTS OF FIELD

-ECTRIC FIELD

- E_t IS CONSERVATING BETWEEN 1 AND 2 $\qquad E_{t_2} = E_{t_1} \qquad E_{x_2} = E_{x_1} = 2$

- D_m IS CONSERVATING BETWEEN 1 AND 2 $\qquad D_{m_2} = D_{m_1}$ $\quad (D = \varepsilon \cdot E)$

$$\varepsilon_2 \cdot E_{y2} = \varepsilon_1 \cdot E_{y1}$$

$$E_{y_2} = \frac{\varepsilon_1}{\varepsilon_2} \cdot E_{y_1} = \frac{2\cancel{\varepsilon_0}}{3\cancel{\varepsilon_0}} \cdot 2 = \frac{4}{3}$$

$$\vec{E_2} = E_{x_2}\vec{u_x} + E_{y_2}\vec{u_y} = \boxed{2\vec{u_x} + \frac{4}{3}\vec{u_y}} \left[\frac{V}{m}\right]$$

MAGNETIC FIELD

- H_t IS CONSERVATING... $\qquad H_{t_1} = H_{t_2} \qquad H_{x_1} = H_{x_2} = 10$

- B_m IS CONSERVATING.. $\qquad B_{m_1} = B_{m_2}$ $\quad (B = \mu \cdot H)$

$$\mu_1 \cdot H_{y_1} = \mu_2 \cdot H_{y_2}$$

$$H_{y_1} = \frac{\mu_2}{\mu_1} \cdot H_{2y} = \frac{2\cancel{\mu_0}}{\cancel{\mu_0}} \cdot 1 = 2$$

$$\vec{H_1} = H_{x_1}\vec{u_x} + H_{y_1}\vec{u_y} = \boxed{10\vec{u_x} + 2\vec{u_y}} \left[\frac{A}{m}\right]$$